Network Performance Model for Location Area Re-Planning in GERAN

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Abstract

In mobile networks, Location Area (LA) planning has a strong influence on network performance. This paper addresses the problem of optimising an existing LA plan in a live GSM-EDGE Radio Access Network (GERAN) based on statistical measurements. Unlike prior work, where the quality of an LA plan was assessed based on the number of location update and paging requests, the main focus in this paper is on estimating the impact of changing the LA plan on the air-interface signalling load. For this purpose, a performance model is proposed to estimate changes in the traffic load in dedicated signalling channels. With such a model, it is possible to estimate the change that a new LA plan will cause in terms of the required number of signalling channels in the whole network, providing an indication of the capacity benefit obtainable, which is a more significant figure of merit for operators. Model assessment is carried out based on measurements taken before and after changing the LA plan of a limited geographical area in a live network. Then, a comparison of several LA re-planning algorithms is performed over measurements in a larger area by using the proposed performance model. Results show that the signalling traffic (and, hence, the required signalling resources) can be significantly reduced by improving the existing LA plan in a live GERAN system.

Key words: mobile network, location area, network optimisation, signalling, graph partitioning

1. Introduction

In recent years, the size and complexity of mobile networks have increased exponentially. One of the basic, yet most time-consuming tasks in operator activities is the mere structuring of the network. Network structure is key to providing an adequate quality of service with minimum infrastructure. This motivates operators to invest much time and effort in optimising network structure frequently. For scalability, mobile networks are given a hierarchical structure. Thus, network structuring aims to find the best clustering of elements in lower layers (often, cells or base stations) to be assigned to controllers (or control areas). Examples of these clustering

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problems are the assignment of base stations to base station controllers [1], packet control units [2], mobile switching centres [3] and location areas [4].

In current networks, the definition of location areas (LAs) has a strong impact on location management processes [5]. The network coverage area is divided into several LAs, each comprising a large number of cells. Any idle terminal crossing an LA boundary performs a Location Update (LU) request (referred to as *mobility LU*). Likewise, idle terminals update their location periodically (referred to as *periodic LU*). In parallel, the network notifies the user about an incoming call via a paging message in all cells belonging to the same LA [6]. With these static procedures, there is a tradeoff between the number of LUs and paging requests when defining the size of LAs. Large LAs lead to more paging traffic, while small LAs lead to more LUs. To overcome this limitation, many dynamic LU and paging schemes have been proposed, which can be adjusted based on user call and mobility patterns. Examples of the latter are the movement-based [7], profile-based [8], timer-based [9], direction-based [9], distance-based [9], predictive distance-based [10] and state-based [11] LU schemes, and the selective [12] and velocity-based [13] paging schemes. Unfortunately, these schemes are not available in existing networks, so operators have to rely on the proper definition of LAs to reduce network signalling.

Like many other clustering problems, the LA planning can be solved as a graph partitioning problem [14]. A first group of heuristic methods make use of graph-theoretic algorithms, namely local refinement [15] and coarsening [4]. A second group of methods formulate the partitioning problem as an integer linear programming model and then use general-purpose optimisation algorithms, either exact (e.g., branch and bound [16]) or heuristic (e.g., genetic algorithms [17, 18, 19], simulated annealing [19, 15], linear programming [20, 21], set covering [22]).

By those methods, an initial LA plan is built during network design from traffic and mobility estimates. Subsequently, due to changes in user trends and the addition of new sites, the initial design might become ineffective, which would require changing the LAs. Another driver to change LAs is the replacement of old and low capacity controllers by new and higher capacity ones. Such an event requires re-allocating sites between controllers, which is an activity that, in a network of the size of an average country, is carried out virtually every day. To ensure that the network graph reflects how users move in the real network, statistics can be obtained from the network management system. These measurement-based methods rely on Hand-Over (HO) statistics for connected users, as current networks do not provide mobility statistics for idle users. Most methods proposed so far aim to minimise the number of HOs between cells in different LAs in the hope that the signalling load is thus reduced. However, experience shows that the number of HOs is not directly linked to the load in signalling channels [23]. Even if this not the case, and HOs and LUs are correlated, the assessment of classical LA planning strategies has only been based on graph partitioning performance indicators (e.g., edge cut, subdomain weight) and very little attention has been paid to the real impact on network operation. Note that the signalling load on the air interface is not exactly proportional to the number of mobility LUs, because the latter is only a fraction of the traffic on dedicated signalling channels, as will be shown later. Thus, it might happen that a large reduction in the number of mobility LUs in a cell did not lead to a significant reduction in the total signalling load in the cell.

This paper provides the link between graph-theoretic and network performance indicators when re-planning LAs in GSM-EDGE Radio Access Network (GERAN). For this purpose, a methodology is proposed to estimate the change that a new LA plan will cause in the required number of dedicated control channels from statistics in the network management system. Such a methodology will enable GERAN operators to predict the benefit that a new LA plan will provide in terms of "saved" time slots before implementing the changes. The proposed methodology combines an idle-user mobility model, a traffic model for dedicated control channels and a dimensioning rule for these channels in GERAN. Model assessment is carried out by analysing the impact of changing the LA plan in a limited geographical area of a live network. Based on the proposed model, a performance comparison of several LA re-planning algorithms is then performed over statistics taken from a larger network area.

To the authors' knowledge, no previous work has investigated the impact of changing the LA plan of a live network. Most previous studies [16, 18, 22] tested their LA planning methods on planar graphs based on a regular grid with hexagonal cells of the same size. The few studies that used real graphs [15][21] evaluated their methods based on graph partitioning indicators and did not check the impact of changes on signalling performance and required signalling resources. The main contributions of this paper are: a) a methodology to estimate the required signalling resources with a new LA plan from measurements in the network management system, and b) a new proof of the correlation between idle and connected user mobility in live networks by showing that changes in inter-LA HOs and LUs are correlated, unlike prior work (e.g., [24][23]), where it was checked that the raw count of inter-LA HOs and LUs were correlated.

The rest of the paper is organised as follows. Section 2 introduces the LA planning problem. Section 3 describes several LA planning methods taken from the graph partitioning literature. Section 4 presents the model to quantify the impact of LA changes on the required signalling resources. Section 5 presents the performance analysis based on measurements taken from a live network. Finally, Section 6 presents the main conclusions of the study.

2. Location Area Planning in GERAN

In this section, the LA planning problem is described from the operator perspective. First, the signalling capacity of current GERAN systems is estimated. Then, the current practice is described and the planning problem is formulated analytically.

2.1. Signalling Capacity in GERAN

In GERAN, signalling capacity largely depends on the capacity of two radio interface channels: the Common Control CHannel (CCCH) and the Stand-alone Dedicated Control CHannel (SDCCH) [6]. The CCCH is made up of four logical channels, amongst which are the Paging CHannel (PCH) and the Access Grant CHannel (AGCH). The PCH is used to broadcast paging messages to notify idle users about an incoming call, while the AGCH is used to assign resources to a user requesting access to the network. The SDCCH is involved in call set-up, mobile station registration and location update, and data services such as Short Message Service (SMS), Multimedia Messaging Service (MMS), Wireless Application Protocol (WAP) and Supplementary Services (SS).

The capacity of the PCH depends on the configuration of the CCCH, the resources assigned to the AGCH and the type of paging message used by the Base Station (BS). Most operators choose a non-combined SDCCH configuration for the CCCH with 1 block reserved for AGCH, while most paging messages in live networks are of Type 2 and 3. In these conditions, the maximum PCH capacity is approximately 400000 individual paging requests per hour [25].

The capacity of the SDCCH depends on the number of Time SLots (TSLs) devoted to it on a per-cell basis. In a non-combined SDCCH configuration, each TSL comprises 8 SDCCH sub-channels. Therefore, the number of SDCCH sub-channels in a cell is a multiple of 8. In some networks, one of these sub-channels is used for the Cell Broadcast CHannel (CBCH), in which case the capacity takes values in the set $8i - 1$, $i \in \mathbb{N}_{+}$.

2.2. *Current Planning Strategy*

The definition of LAs is Mobile Switching Centre (MSC) internal functionality. In the ideal LA plan, the capacity of the PCH determines the size of the LA, i.e., how many users can be served by an LA. At the same time, the shape of LAs should ensure that BSs exchanging lots of users are assigned to the same LA so that the number of mobility LUs is minimised. Thus, the total SDCCH traffic is reduced and the required SDCCH capacity in terms of TSLs is reduced. Minimising the SDCCH TSL requirements is desirable as each TSL devoted to carry mobility LU traffic cannot be used to carry revenue generating user traffic. Hence, schemes to optimise LA plans are of direct benefit to operators. In these conditions, it is the change in the SDCCH TSL requirements that ultimately relates to the benefit of a new LA plan as it indicates how many SDCCH TSLs can be converted from carrying signalling traffic to carrying actual payload.

For simplicity, operators have traditionally defined LAs by assigning whole Base Station Controllers (BSCs), and not individual cells, to LAs. Thus, the LA plan has BSC resolution, i.e., all cells in the same BSC are assigned to same LA. Also for simplicity, an LA seldom consists of more than one BSC. Such an approach does not take full advantage of LA planning. On the contrary, with the limited size of current BSCs, many LAs could be merged without exceeding the maximum paging capacity of cells in the LA.

To cope with the increased LU traffic from non-optimal LA plans, operators have to re-plan the number of SDCCH TSLs on a cell-by-cell basis relatively frequently. For this purpose, a very conservative approach is followed to ensure SDCCH blocking ratios well below 1%. In those cells where SDCCH congestion occurs, new SDCCH TSLs are added. Such an approach is equivalent to dimensioning SDCCH capacity for the peak traffic. As a result of this reactive approach, SDCCH resources are often over-dimensioned in live networks [26]. Thus, SDCCH resources can be reduced in many cells without affecting network performance. This reduction can be even larger if the peak SDCCH traffic in cells is reduced by improving the current LA plan. Note that peak SDCCH traffic in many cells on LA borders is due to LUs from users moving in groups, which could be eliminated by a proper LA plan.

2.3. *Problem Formulation*

The definition of LAs is formulated here as a classical graph partitioning problem [14]. In this approach, the optimised network area is modeled by a non-directed weighted graph. The vertices of the graph represent the cells, while the edges represent the adjacencies defined by the operator for HO purposes. The weight of each vertex, ω_i, denotes the number of paging requests due to mobile terminated calls in the cell, while the weight of each edge, γ_{ij}, denotes the number of idle users moving between cells on its ends. The latter can be estimated from HO statistics, as suggested in [23] and [24] and will also be confirmed later. The former must be estimated from counters of paging requests and mobile terminated calls.

The partitioning of the graph, performed by grouping vertices into disjoint subsets, referred to as *subdomains*, reflects the assignment of cells to LAs. The resulting partition defines a set of edges that join vertices in different subdomains, δ, referred to as a *cut*. Formally, the LA planning problem can be modeled as a *bounded, min-cut problem*, [27], described as follows. Let G be an undirected weighted graph $G = (V, E)$, consisting of a set of vertices V and edges E, vertex weights ω_i and edge weights γ_{ij}. Let B_{aw} be a real number representing the maximum weight of a subdomain. The problem stands for the partition of V into an arbitrary number of subdomains k, $S_1, S_2, \cdots, S_k$, such that the sum of the weight of vertices in each subdomain is less than B_{aw} and the sum of the weights of edges in the cut, referred to as *edge cut*, is minimised.

It should be pointed out that the edge cut does not include any information about the weight of subdomains, representing the paging load in the network, which has only been considered as a constraint. Such a decision is aligned to the way most operators define the LA planning problem, where it is assumed that the cost of an LU request in terms of signalling is much larger than a paging request (e.g., 17 times larger in [28], 10-30 times in [21]). An additional benefit of not considering that term is the possibility of reusing classical graph partitioning algorithms (which only consider the edge cut in the objective function), whose behaviour is well known and for which very effective codes exist in the public domain.

In the past, LAs could not cross MSC boundaries. Hence, an LA could only comprise BSCs in the same MSC and LA planning had to be done on a per-MSC basis. Thus, a whole operator network comprised several instances of the problem (one per MSC). Nevertheless, it is common that several MSCs and BSCs are co-located and, therefore, the physical ”re-wiring” of a BSC between co-located MSCs can be carried out with relatively little effort or investment. In addition, current equipment allows multiple physical MSC Server Systems to be logically grouped into one MSC Server System. Therefore, it is now possible that some LAs cross MSC borders. Thus, LA planning can be performed network wide instead of on a per-MSC basis and LA re-planning can be done without any change in the transmission network.

3. Solution Algorithms

The *bounded, min-cut problem* is known to be NP-complete [29] and, in general, it is not possible to compute optimal partitionings for graphs of interesting size. Thus, several efficient heuristics have been proposed in the literature (for a survey, see [14]). However, unlike in other applications, the size of graphs is small when LA planning is performed with BSC resolution. In this case, exact approaches are feasible. The following paragraphs outline four graph partitioning algorithms to re-plan LAs, which are the common benchmark in the graph partitioning literature.

3.1. Exact Method

As shown in [3], the graph partitioning problem can be formulated as the Integer Linear Programming (ILP) model

$$\text{Min} \quad \sum_{(i,j)\in E} \gamma_{ij}(1 - \sum_{n\in N} Z_{ijn}) \tag{1}$$

$$\text{s.t.} \quad \sum_{n\in N} X_{in} = 1, \qquad \forall\, i \in V, \tag{2}$$

$$\sum_{i\in V} \omega_i X_{in} \le B_{aw}, \qquad \forall\, n \in N, \tag{3}$$

$$Z_{ijn} \le X_{in}, \qquad \forall\, (i,j) \in E,\ n \in N, \tag{4}$$

$$Z_{ijn} \le X_{jn}, \qquad \forall\, (i,j) \in E,\ n \in N, \tag{5}$$

$$Z_{ijn} \ge X_{in} + X_{jn} - 1, \quad \forall\, (i,j) \in E,\ n \in N, \tag{6}$$

$$X_{in} \in \{0,1\}, \qquad \forall\, i \in V,\ n \in N, \tag{7}$$

$$Z_{ijn} \in \{0,1\}, \qquad \forall\, (i,j) \in E,\ n \in N, \tag{8}$$

where X_{in} and Z_{ijn} are binary variables that reflect the assignment of cell i and adjacency (i,j) to LA n, respectively, and $N = \{1, 2, \cdots, k\}$, where k is the number of LAs. (1) reflects the goal of minimising the number of HOs between cells in different LAs. (2) ensures that a cell belongs to only one LA. (3) reflects the paging capacity limit of cells in an LA, B_{aw}. (4)-(6) show the dependence between decision variables by linear constraints and (7)-(8) are binary constraints. The model (1)-(8) can be solved exactly by the *Branch-and-Cut* (BC) algorithm [30].

It should be pointed out that the value of k (i.e., the number of LAs) in the optimal solution is not known a priori. In this work, k for this method is fixed to the value in the best heuristic solution, which is computed by one of the method described next.

3.2. Heuristic Methods

3.2.1. Local Refinement

The simplest approach to partition a graph is by improving an existing solution, for which the *Fiduccia and Mattheyses* (FM) refinement algorithm [31] can be used. The FM algorithm progressively improves an initial partition by checking all possible movements of every single vertex to other subdomains and selecting that yielding the greatest edge-cut reduction. The main strength of the algorithm is its ability to escape from local minima by exploring movements that temporarily increase the edge cut.

Unlike classical refinement algorithms, the FM algorithm used in this work can reduce the number of subdomains in the initial solution, i.e., subdomains can become empty after moving a vertex. Hence, it is not possible to know a priori the number of subdomains in the final solution. This behaviour is also observed in the algorithms described next, since the latter include local refinement algorithms to improve their partitions.

3.2.2. Random Greedy Graph Growing Partitioning

When the initial solution is of poor quality, it is advantageous to build the partition from scratch. In the *Greedy Graph Growing Partitioning* (GGGP) algorithm [27], k initial seed vertices are chosen. Then, a partition of the graph is built by growing subdomains incrementally around arbitrary seed vertices. At each step, the growing region with the smallest weight is selected. Then, the algorithms adds the unassigned vertex with the heaviest edge weight to vertices already in the growing region. To improve the robustness of the method, a limited number of trials can be conducted with randomly selected seed vertices [32]. Such an approach is referred to as *Random GGGP* (RGGGP). The diversity provided by random seed selection proves to be extremely valuable in graphs with few vertices per subdomain (as will be shown to be the case of LA planning with BSC resolution).

Again, note that the optimal value of k is not known a priori. Thus, several trials of RGGGP with different values of k must be performed.

3.2.3. Multi-Level Refinement

In the Multi-Level (ML) approach [33], the original graph is coarsened by collapsing vertices together. Then, an initial solution is efficiently computed on the coarsened graph. Finally, the solution is progressively uncoarsened to obtain the partition of the original graph. After each uncoarsening step, a local refinement algorithm (e.g., FM) is applied on small portions of the graph close to the partition boundary. This technique avoids the problem of local minima in local refinement algorithms, improving both solution quality and runtime.

The ML algorithm in this work coarsens the graph until the number of vertices in the graph coincides with the number of subdomains [34]. In each coarsening step, coarsening is performed by selecting the heaviest edges on the graph [35]. In the initial partitioning, different values of k must be tested, since the optimal value of k is not known a priori.

4. SDCCH Performance Model

Before modifying an existing LA plan, operators need to know the exact impact of changes on SDCCH performance. Likewise, operators might want to quantify the number of SDCCH TSLs that could be converted into TSLs for payload traffic if the new LA plan was implemented. In this section, a model is proposed to estimate the change that a new LA plan will cause in terms of the number of required SDCCH TSLs. For this purpose, the peak traffic during the Busy Hour (BH) with the new plan is estimated on a cell basis from statistics in the network management system. Such a peak value gives the number of required TSLs due to the conservative rule used by operators to plan SDCCH resources.

The input data consists of HO statistics, number of paging requests, number of SDCCH attempts classified per cause[1], mean SDCCH holding time and average and peak SDCCH traffic with the old plan. Ideally, all statistics should be collected on a cell basis, except HOs, which must be on an adjacency basis. However, current BSCs only reflect the sum of paging requests directed to users in a location area, since the paging algorithm is MSC functionality. Thus, all cells in an LA have the same value of paging traffic. From statistics of mobile terminated calls, available on a cell basis, it is possible to estimate the number of paging requests originated per cell. Likewise, all measurements should be collected on the SDCCH BH. Unfortunately, most operators only gather HO statistics on a daily basis for storage reasons. In this case, some transformation between daily and BH figures are needed. In addition, measurements are collected for several days and weekly averages are performed to increase the robustness of estimates.

The model starts by computing the difference in the daily sum of *incoming inter-LA HOs* (IIHOs) (i.e., HOs coming from cells in other LAs) per cell. For this purpose, a directed graph is built from HO statistics. This graph differs from the one used by graph partitioning algorithms, where incoming and outgoing adjacencies are aggregated into a single bi-directional adjacency for computational efficiency. Then, the sum of edge weight incident to each vertex from edges in the cut is computed for the old and the new plan. By subtracting both quantities, the expected difference in the total daily sum of IIHOs in cell i, $\Delta\widehat{IIHO}_{day}(i)$, is obtained as $\Delta\widehat{IIHO}_{day}(i) = \widehat{IIHO}_{day}(i) - IIHO^{(0)}_{day}(i)$, where $\widehat{IIHO}_{day}$ and $IIHO^{(0)}_{day}$ are the values with the new and the old plan, respectively.

Second, the difference in the total daily sum of LUs, $\Delta\widehat{LU}_{day}$, is estimated per cell as

$$\Delta\widehat{LU}_{day}(i) = \widehat{c}_{LU/IIHO}\ \Delta\widehat{IIHO}_{day}(i)\,, \tag{9}$$

where $\widehat{c}_{LU/IIHO}$ is the ratio of idle to connected users moving, computed by linear regression between LU and IIHO statistics. In (9), it is assumed that: a) mobility LUs are proportional to IIHOs, and b) the number of periodic LUs does not change with LA planning. Both assumptions will be shown to be valid in the next section. Note that, unlike prior studies [24][23], (9) works on differential rather than absolute measurements (i.e., comparing changes instead of raw figures). Thus, the influence of periodic LUs is eliminated, since these do not change with LA re-planning, and there is no need for estimating this part of LUs explicitly as in [23]. Also important, any relationship between IIHOs and LUs through a third variable (e.g., cell population) is reduced. An example of such a phenomenon (known as confounding [36]) would be the case of a network where hot-spot cells showed lots of IIHOs and LUs just because of a high cell population, but

[1] SDCCH establishment causes are: Mobile Originated Call, Mobile Terminated Call, Emergency Call, Call Re-establishment, Location Update, IMSI Detach, Supplementary Service, Short Message Service and Ghost Seizure.

LUs were mostly due to adjacent cells with few outgoing IIHOs. Checking the changes in IIHOs and LUs after re-defining inter-LA adjacencies reduces the possibility of confounding.

Third, daily LU figures are translated into BH LU figures. Here, it is taken into account that LUs are not distributed the same in all cells during a day. In some cells, all LUs take place during the BH, while in other cells LUs are evenly distributed along the day. Thus, the difference in the number of LUs in the BH is computed as

$$\Delta\widehat{LU}_{BH}(i) = \widehat{c}_{BH/day}(i)\ \Delta\widehat{LU}_{day}(i)\,, \tag{10}$$

where $\widehat{c}_{BH/day}(i)$ is a constant between $\frac{1}{24}$ and 1 showing the BH-to-daily ratio of LU traffic in cell i obtained from measurements. From this value, the difference in the average SDCCH carried traffic (in Erlangs) can be calculated as

$$\Delta\widehat{A}_{cBH}(i) = \frac{\Delta\widehat{LU}_{BH}(i)\ MHT_{LU}}{3600}\,, \tag{11}$$

where MHT_{LU} is the mean holding time of LU attempts in the network. Note that MHT_{LU} is not measured and can greatly vary from network to network. Therefore, instead of using values reported in the literature (e.g., [25]), a multiple regression analysis is performed on counters of SDCCH attempts per cause and mean SDCCH holding time in cells during the BH, as

$$A_c(i,h) = \sum_c \widehat{\beta}_c \frac{N_c(i,h)}{3600} + \epsilon(i,h) \quad \forall\ i,h, \tag{12}$$

where $A_c(i,h)$ and $N_c(i,h)$ are the average SDCCH traffic and number of SDCCH BH attempts for cause c in cell i and hour h, respectively, $\widehat{\beta}_c$ is the regression coefficient for cause c, and $\epsilon(i,h)$ is the residual in cell i and hour h. In particular, $\widehat{\beta}_{LU}$ is the least-squares estimate of MHT_{LU} for the entire network. From (11), the new average SDCCH traffic, $\widehat{A}_{c_{BH}}$, can be easily computed from the old value, $A^{(0)}_{c_{BH}}$, as

$$\widehat{A}_{c_{BH}}(i) = A^{(0)}_{c_{BH}}(i) + \Delta\widehat{A}_{c_{BH}}(i) = A^{(0)}_{c_{BH}}(i) + \frac{\Delta\widehat{LU}_{BH}(i)\,\widehat{\beta}_{LU}}{3600}\,. \tag{13}$$

To convert average traffic values into peak traffic values, a regression curve is built from network statistics. To explain the reason, Fig. 1 depicts a scatter plot of average versus peak SDCCH traffic in cells of a live network. Each point corresponds to a measurement in a cell and hour. It is observed that peak traffic values are highly variable, ranging from 0 to 60. More unexpected, for the same value of average traffic, many different peak values are possible. Such a behaviour is not only due to the stochastic nature of sampling, but rather to the time correlation between SDCCH attempts in some cells. In [37], it was shown that, in some cells, LU attempts, which are more than half of the SDCCH traffic in real networks [38], tend to concentrate in a short period of a time. Such a phenomenon is due to users moving in public transport crossing an LA border together. At this point, it is worth noting that the aim of the study is to quantify the overall impact of a new LA plan on a long-term basis, and not the impact on every single cell at all times. Thus, a regression curve, $\widehat{f}_{pk2av}$, is derived to reflect the main trend. Although the value of the coefficient of determination is small (i.e., $R^2 = 0.16$), it is expected that this lack of fit does not affect global estimates when aggregated network wide for several days. Even if a certain cell might experience large deviations from the estimated peak value, such deviations should cancel

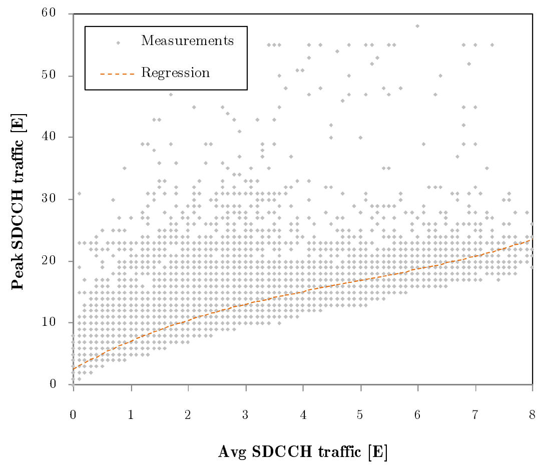


Figure 1: Relationship between average and peak SDCCH traffic in a live network.

out when all cells affected by LA changes are taken into account. By means of this curve, the expected difference in the peak value is computed as

$$\begin{aligned} \Delta\widehat{A}_{pk_{BH}}(i) &= \widehat{A}_{pk_{BH}}(i) - \widehat{A}^{(0)}_{pk_{BH}}(i) \\ &= \widehat{f}_{avg-pk}(\widehat{A}_{c_{BH}}(i)) - \widehat{f}_{avg-pk}(\widehat{A}^{(0)}_{c_{BH}}(i)) \,. \end{aligned} \tag{14}$$

where $\widehat{f}_{avg-pk}(*)$ denotes the value obtained by the regression curve.

Finally, the difference in the number of required SDCCH TSLs is obtained. A rough estimate could be made by dividing $\Delta\widehat{A}_{pk_{BH}}$ by 8, since 1 SDCCH TSL comprise 8 subchannels. A more accurate estimate is obtained by considering that, due to the granularity given by the TSL assignment, a) 1 SDCCH TSL is saved per each multiple of 8 that falls within the interval $[\widehat{A}_{pk_{BH}}, \widehat{A}^{(0)}_{pk_{BH}}]$, even if $\Delta\widehat{A}_{pk_{BH}} < 8$, and b) a minimum of 1 SDCCH TSL is required in the non-combined configuration. Thus, the number of saved TSLs, $\Delta\widehat{N}_{SDCCHTSL}$, is estimated as

$$\Delta\widehat{N}_{SDCCHTSL}(i) = \min\left\{1, \left\lceil \frac{\widehat{A}_{pk_{BH}}(i)}{8} \right\rceil\right\} - \min\left\{1, \left\lceil \frac{\widehat{A}^{(0)}_{pk_{BH}}(i)}{8} \right\rceil\right\} \,. \tag{15}$$

It is worth noting that classical queueing models fail to give accurate estimates of SDCCH blocking. In [37], it was shown that, due to retrials and time correlated arrivals, Erlang-B and C formulas tend to under-estimate SDCCH blocking in live networks. More importantly, burstiness greatly varies from cell to cell, which makes tuning of analytical models very difficult. This is the main reason for not using an analytical model to estimate the number of required channels from average traffic values, and using peak traffic instead.

5. Performance Analysis

The proposed performance model is validated from measurements taken before and after changing the LA plan of a live GERAN. Then, the LA planning algorithms described in Section 3 are applied to a graph constructed with real data. These algorithms are compared using the proposed SDCCH performance model. The aim of the analysis is to: a) check the validity of the SDCCH performance model, b) identify the best LA planning algorithm among those in the graph partitioning literature, and c) quantify the reduction in the number of TSLs from an optimal LA plan in a live network. For clarity, model assessment is presented first and method assessment is then discussed. In both cases, the preliminary conditions are described first and subsequently the performance results are discussed.

5.1. Model Assessment

To check the model, the location area plan of a live network had to be modified. Since operators are not willing to change network settings, the analysis had to rely on changes made by the operator for other purposes. The analysis presented here was based on changes caused a BSC-splitting event. Every re-assignment of a BS to a different BSC causes a change in the LA plan due to the way LAs are currently assigned, where almost every BSC has its own LA. By checking differences in SDCCH traffic in affected cells, it was possible to check model accuracy.

It should be pointed out that BSC-splitting events are infrequent and only take place locally, which is the reason for selecting a limited area for checking the model. Nonetheless, it is expected that the resulting model is accurate enough to raise global estimates for the whole network, since the model is flexible enough to consider all local variables having an impact on SDCCH performance and the restricted area comprises a variety of environments ranging from dense urban to open rural areas.

5.1.1. Analysis Set-up

The network area consists of 519 BSs, distributed over 177 sites, covering a geographical area of 4000 km^2. Initially, these BSs were controlled by 4 BSCs, each representing a different LA. As a result of BSC splitting, a new BSC was added and 77 BSs of the old BSCs were re-assigned to new BSC, causing that 196 BSs experienced changes in their incoming inter-LA adjacencies. In these BSs, measurements were gathered one week before and after implementing the changes. During the analysis, differential measurements are computed by comparing the same day of the week in the before and after periods (note that the BH might not be the same). Specifically, the difference was defined as the value with the new plan minus the value with the old plan. To avoid the influence of unpredictable events, some isolated measurements with abnormal conditions of SDCCH availability or excessive change in daily call attempts were discarded.

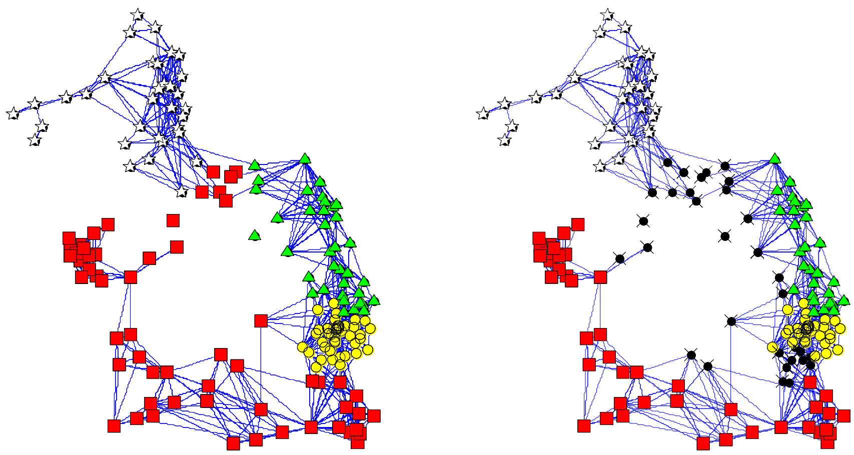
Fig. 2 (a)-(b) show the old and new LA plans of the area, respectively. In the figure, a symbol represents the physical location of a BS. Note that all BSs in a site are represented in the same location. Each of the 4-5 different symbols represent a different LA (i.e., BSC). Lines represent adjacencies between BSs. From the figure, it can be understood that the analysis focuses on BSs adjacent to those re-allocated, represented by an 'x' symbol in Fig. 2 (b).

The model is validated by comparing estimates of the peak SDCCH traffic in the BH with the new LA plan from statistics with the old plan. To cope with the high variability of peak traffic measurements, a weekly average of estimates is performed, resulting in a single value per cell. Thus, more robust estimates are obtained.

5.1.2. Analysis Results

First, the different steps in the model are justified and subsequently the overall performance is evaluated.

Fig. 3 (a)-(c) show the relationship between changes of different network performance indicators, as they are used in the model (i.e., (9), (10) and (11), respectively). Each point in the figures



(a) Old plan

(b) New plan

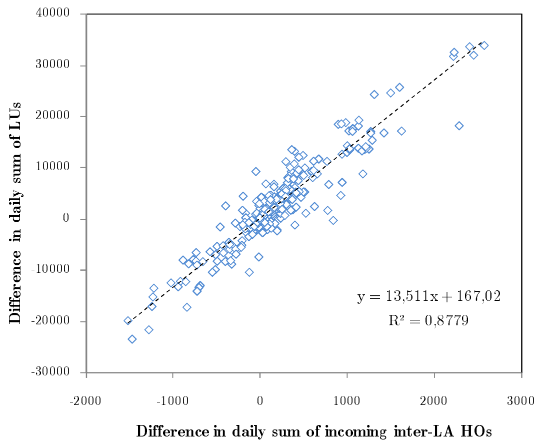
Figure 2: Location area plans before and after BSC splitting.

represent a day of the week in a cell. A regression line is superimposed, together with the value of the coefficient of determination, R^2.

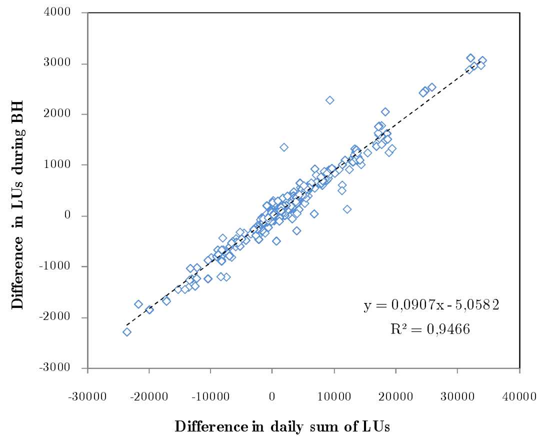
Fig. 3 (a) shows the relationship between changes in IIHOs and LUs. First, it is observed that most samples have positive values. This is due to the fact that, by changing from 4 to 5 LAs, the number of inter-LA adjacencies in the area increased by 43%. Specifically, the number of IIHOs and LUs in the 196 cells affected by changes increased by 27% and 17%, respectively. From the large value of R^2 in the figure, it can be deduced that IIHOs and LUs are strongly correlated. This is proof of the correlation between idle and connected mobility statistics. By using differential measurements, it is expected that only mobility LUs (and not all types of LUs) are included in the comparison. From the line equation, it can be inferred that there are 13.5 idle users per connected user moving.

Fig. 3 (b) and (c) show the relationship between changes in the daily sum of LUs, BH LUs and average SDCCH traffic. In the latter indicator, the variation of SDCCH traffic due to attempts other than LU has been subtracted. This term is calculated from attempt counters in the before and after periods and the mean holding time on a per-service basis, obtained from (12). From the figures, it can be concluded that the three indicators are highly correlated. Note that the slope of the regression line in Fig. 3 (c) gives a rough estimation of the value of $MHT_{LU}/3600$. The value of MHT_{LU} thus obtained (i.e., 4.3 seconds) coincides with β_{LU} in (12). It is worth noting that such a tight link does not exist between changes in the average and peak traffic, since the absolute values of these variables are related by a non-linear function (i.e., f_{pk2av}). Therefore, similar changes in the average traffic can lead to totally different changes in peak traffic.

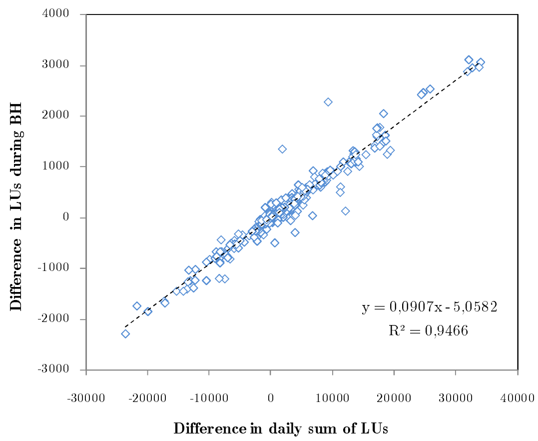
Hitherto, the analysis has justified the steps in the estimation process. The last figure shows the overall accuracy of the model. Fig. 4 (a)-(b) shows a scatter plot of estimates versus measurements for weekly averages of the difference in average and peak SDCCH BH traffic. From Fig. 4 (a), it can be concluded that the model can estimate changes in the average SDCCH traffic caused by a new LA plan quite accurately on a cell basis. However, Fig. 4 (b) shows that the model only gives a rough approximation of changes in the peak traffic. In particular, 2 outliers



(a)

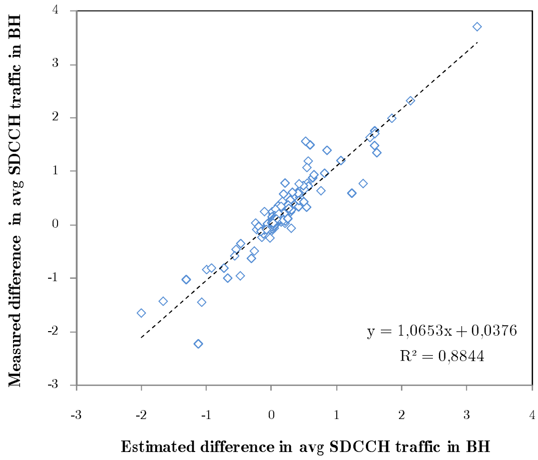


(b)

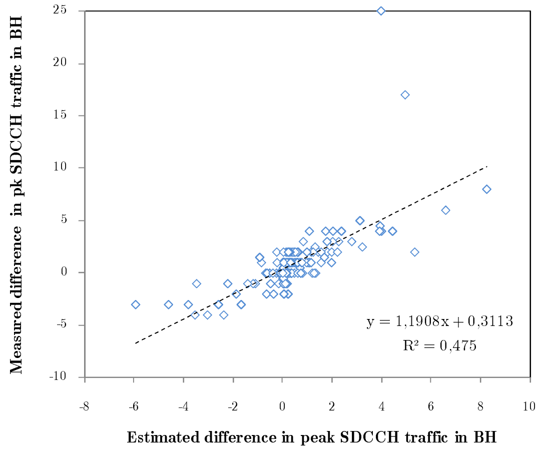


(c)

Figure 3: A scatter plot of changes in several performance indicators.



(a) Average SDCCH traffic



(b) Peak SDCCH traffic

Figure 4: A scatter plot of measurements versus estimates of changes in SDCCH traffic.

are clearly identified. A closer analysis shows that these two cells face each other at both sides of an LA border crossed by a high-speed railway line. If those outliers are removed, R^2 is 0.67, which is considered high enough for network-wide estimations.

5.2. *Planning Method Assessment*

The above-described model is used to compare the performance of several LA planning methods over real graphs. For this purpose, a larger geographical area is considered.

5.2.1. *Analysis Set-up*

The extended network area covers 100000 km^2, comprising 9807 BSs, 3353 sites, 76 BSCs, 20 MSCs and 71 LAs. Statistical performance measurements were gathered for the whole area during 2 weeks. Table 1 shows some statistics of the area. From the table, it can be deduced that the number of TSLs devoted to SDCCH is 7% of total network capacity. An upper bound of the number of saved TSLs achieved by optimising the LA plan is the number of SDCCH TSLs in BSs with more than 1 TSL minus the minimum number of SDCCH TSLs in these BSs (i.e., 6902-3237=3665). Likewise, it is observed that the maximum paging load per LA (i.e., 134959) is far from the maximum capacity value (i.e., 400000). This evidence clearly indicates that the size of LAs can be made at least three times larger without compromising paging performance. From the sum of paging load, it can be deduced that the minimum number of LAs is 14.

Although the area comprised several MSCs, a single instance of the LA planning problem is considered, as it is assumed that the core network includes the feature that allows LAs to cross MSC borders. Nonetheless, the results should be representative of a typical network, as the graph is built from real data and the size of the instance is several orders of magnitude larger than in previous studies [15][21]. The network graph is built from statistics collected during 2 weeks. Edge weights are computed by dividing the number of HOs in 2 weeks by 14 to obtain the daily average. Vertex weights are the maximum number of paging requests originated in cells during

Table 1: Statistics in the extended scenario with existing configuration.

Total nbr. of BSs	9807
Total nbr. of TSLs	202240
Total nbr. of SDCCH TSLs	13472
Nbr. of BSs with $N_{SDCCHTSL} > 1$	3237
Total nbr. of SDCCH TSLs in BSs with $N_{SDCCHTSL} > 1$	6902
Maximum BH paging load per LA [att./h]	134959
Total sum of BH paging load [att./h]	5208060
Total sum of BH avg. SDCCH traf. [E]	11294.2
Total sum of pk. BH SDCCH traf. [E]	71300

the SDCCH BH in 2 weeks. As an optimisation constraint, the sum of paging requests in an LA must be below 400000 unless stated otherwise (i.e., B_{aw}=400000).

During the analysis, several methods are compared. To quantify the improvement of a new LA plan, the initial operator solution (denoted as OI for Operator Initial) is evaluated first. Then, several graph partitioning algorithms are tested: a) the BC algorithm over the ILP model of the problem (denoted as BC), b) the FM refinement of the operator solution (denoted as FM), c) the ML refinement algorithm (denoted as ML), and d) the RGGGP algorithm, based on the repeated use of GGGP with random seed selection and FM refinement. The exact method uses the BC algorithm in SCIP [39], whereas the heuristic methods were implemented from scratch in Matlab [40]. In the latter, the number of passes in the FM refinement algorithm is 4. All methods are tested with different target values of k, ranging from 14 to 20, and the best solution is selected. Note that FM and ML are deterministic and, therefore, a single run is performed per value of k. In contrast, RGGGP is randomised and hence produces a different solution for each different random seed. Reported values for RGGGP correspond to the best solution found in 100 attempts per value of k.

During the comparison, heuristic methods are applied to the graphs of site and BSC resolution. The exact method is only tested on the graph of BSC resolution for computational efficiency. Performance assessment is based on solution quality in terms of edge cut. By using the proposed performance model, edge-cut figures are translated into changes in SDCCH traffic and saved TSLs. Time efficiency is not considered in the analysis as it is not a critical issue, since LA re-planning is performed, at most, once a week.

5.2.2. Analysis Results

Table 2 presents the results obtained by the methods on the graph of the extended area. For clarity, the methods have been grouped by the granularity of the graph (i.e., BSC or site). For each method, the edge cut, the maximum subdomain weight and the number of subdomains are shown (i.e., total daily sum of IIHOs, maximum BH paging load and number of LAs, respectively). From the table, it is clear that, regardless of graph resolution, any of the algorithms can reduce the edge cut of the initial operator solution by fully exploiting the paging capacity limit. On the BSC-level graph, RGGGP and BC are the best methods in terms of edge cut. Not shown is the fact that RGGGP finds the optimal solution in minutes while BC takes hours. Specifically, RGGGP can reduce the edge cut of the initial solution by 73%. Its superiority over other heuristics is due to the small number of vertices per subdomain, which makes balancing subdomain weights more complicated. In this situation, the diversity provided by random seed selection is extremely

Table 2: Performance of partitions built by methods.

Method	Resolution	Edge cut $[\cdot 10^6]$	Max. subd. wgt.	Nbr. of subd.
OI	BSC	29.66	134959	71
FM	BSC	8.35	394896	16
ML	BSC	8.69	395508	16
RGGGP	BSC	7.94	394896	15
BC	BSC	7.94	394896	15
FM	Site	20.21	126224	70
ML	Site	4.13	397076	16
RGGGP	Site	7.87	384020	15

Table 3: Estimated SDCCH performance of best heuristic methods.

Method	Resolution	Avg. SDCCH BH traf. [E]	Diff. Avg. SDCCH BH traf. [E]	Peak SDCCH BH traf. [E]	Diff. peak SDCCH BH traf. [E]	Nbr. of SDCCH TSLs	Diff nbr. of SDCCH TSLs
OI	BSC	11294.2	-	71300.0	-	13597	-
RGGGP	BSC	8667.9	-2626.3	61814.1	-9485.9	12638	-959
ML	Site	8167.2	-3127.0	58945.8	-12354.2	12368	-1229

valuable. In contrast, on the site-level graph, ML outperforms FM and RGGGP. This is due to the fact that, as the graph gets larger, the solution space becomes larger and the optimisation surface has many local minima. Nonetheless, the surface has a globally convex structure, where the best local minima are clustered in a central position of the solution space. Thus, FM and RGGGP get trapped in the local minima. Multiple trials in RGGGP do not alleviate this problem, as random local minima in the graph partitioning problem tend to all have average quality and little variance [41]. This problem is solved in ML by coarsening the graph first. Thus, the optimisation surface is smoothed by eliminating many local minima, which makes local refinement algorithms more effective [34].

In Table 2, it is assumed that the maximum paging channel traffic, B_{aw}, is 400000 messages per hour. Fig. 5 shows the influence of B_{aw} on the edge cut obtained by the methods. As expected, the edge cut increases as B_{aw} decreases, since fewer vertices can be grouped into the same subdomain, leading to more subdomains and edges joining vertices in different subdomains. Roughly, the edge cut is doubled when B_{aw} changes from 400000 to 200000. Nonetheless, the edge cut is still much lower than that of the initial operator solution.

The rest of the analysis is focused on the best heuristic methods, i.e., RGGGP with BSC resolution and ML with site resolution. Table 3 presents the expected impact of the new plans on SDCCH performance. The performance of the old LA plan is also included in the table. For a fair comparison, the total number of SDCCH TSLs with the existing plan is estimated as in the other methods (i.e., by the sum of peak traffic in the area during the BH), instead of using the value currently configured by the operator, reported in Table 1 (i.e., 13597 instead of 13472). Results show that re-planning LAs can reduce the average SDCCH BH traffic by up to 28%. Note that this is much less than the edge cut reduction shown in Table 2 (which has been used in the literature to assess LA planning methods). Most of the benefit (i.e., 85%) is already obtained by the solution with BSC resolution. The reduction in the peak SDCCH traffic is not so large

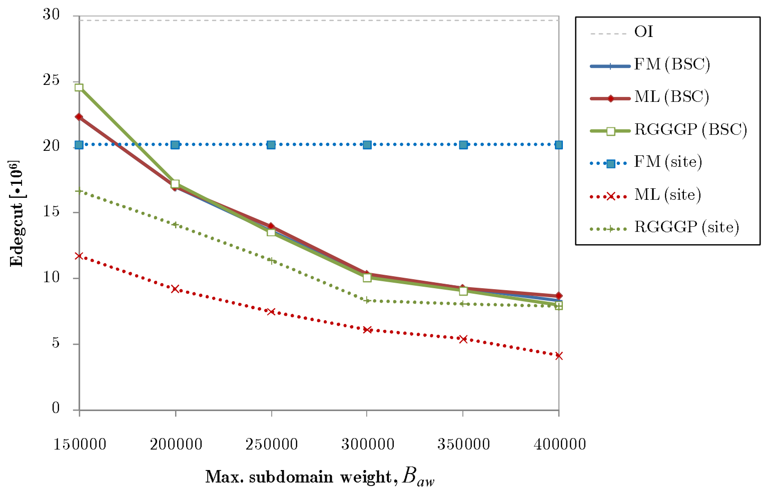


Figure 5: Influence of maximum subdomain weight on edge cut.

(i.e., 17%). This is due to the fact most changes occur in cells with a large average traffic value, where sensitivity to changes is lower, as shown in Fig. 1. Nonetheless, the reduction in peak SDCCH traffic is noticeable. Finally, the number of "saved" TSLs is only 10%. This is partly due to the fact that only 33% of the BSs have more than 1 TSL. Specifically, the best of LA plans (i.e., ML with site resolution) can obtain 33% of the maximum achievable reduction reported in Table 1 (i.e., 3665). This figure is remarkable, as LU traffic is only 50% of the SDCCH traffic in live networks [38].

6. Conclusions

In future cellular networks, higher traffic demand and smaller cell size will make network structuring a complex issue. In parallel, changes in traffic and mobility trends and the addition of new network elements will force operators to re-configure network structure frequently, which can only be done automatically. For optimal performance, automatic structuring methods should use accurate mobility models and effective clustering algorithms. In this paper, the problem of optimising an existing LA plan in a live GERAN system based on statistical measurements has been addressed. Unlike previous work, the main focus has been on estimating the impact of changes in the LA plan on the signalling load. For this purpose, a network performance model has been proposed to estimate changes in the peak traffic on the SDCCH. Model assessment has been carried out based on measurements before and after changing the LA plan of a limited area during a BSC splitting event. Based on the proposed model, a comparison of several LA re-planning algorithms inspired in graph theory has been carried out over graphs built from measurements in a larger area.

Results show that the proposed model can estimate changes in the average SDCCH traffic caused by a new LA plan accurately, while giving adequate estimates of changes in the peak SDCCH traffic. Based on this model, it has been shown that the average SDCCH traffic can be reduced by up to 28% by proper planning of LAs. However, such a reduction only implies a reduction in the required number of SDCCH TSLs by 10%. Hence, it can be concluded that the benefit from re-planning LAs is reducing SDCCH blocking, rather than assigning additional resources for payload traffic.

The proposed model is for the SDCCH, which is only used by idle users. Signalling from/to connected users is carried through the SACCH instead. SACCH carries signalling messages associated to an on-going connection and paging messages for an incoming SMS or call, since the MSC knows the BSC and cell where the user is located. Note that all messages in the SACCH correspond to an individual user and do not depend on how LAs are defined. Therefore, changes in the LA plan will not have an influence on SACCH performance. Likewise, no LU procedure is triggered while the terminal is in busy mode, since the network knows the location of the connected terminal with cell resolution. If a connected user crosses an LA border, the LU is triggered right after the end of the connection, when the user returns to idle mode. Such an LU request is performed on the SDCCH. Thus, LU statistics should include all LA crossing events, both of idle and connected users (provided that a user does not cross several LA borders during a call). Even if this is not the case, it is the SDCCH performance what operators are more interested in. Also note that the ratio of connected to idle users is small (e.g., 1/16 in [23]).

Although the models and methods presented here have been conceived for GERAN, they can be easily extended to other radio access technologies. Future work is to consider more sophisticated SDCCH traffic models including retrials and correlated arrivals that can be tuned on a cell basis [37].

7. Acknowledgements

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