Simulating Oligopolistic Pool-Based Electricity Markets: A Multiperiod Approach

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***Abstract*—This paper simulates a pool-based electricity market and identifies equilibrium patterns. A multiperiod framework that requires the use of continuous and binary variables is considered. Producers can be either price-takers or price-makers, although price-makers and their behavior characterize the market. The elasticity of the loads is modeled through piecewise constant price-demand curves. The market operator uses a detailed network-constrained market-clearing auction that results in hourly market-clearing prices. The behavior of the market participants and the market itself are characterized through a repetitive simulation procedure. The tool presented in this paper is particularly useful for the market regulator that may use it to monitor the market and to identify the exercise of market power by producers. Results from a realistic case study are presented and discussed.**

***Index Terms*—Market equilibrium patterns, market power, multiperiod market simulator, pool-based electric energy markets.**

NOMENCLATURE

The notation used throughout the paper is described below.

A. Indices

i	GENCOs in the market.
t	Time periods considered in the time horizon.
n, k	Network nodes.
j	Generating units.
d	Demands.
h	Power blocks for each demand.
b	Power blocks for each generating unit.

B. Functions

$\lambda_t^i(q_t^i)$	Stepwise monotonically decreasing discontinuous function that expresses the market-clearing price as a function of the quota of GENCO i for hour t. This function is known as price-quota curve.
c_{jt}	Production cost for hour t of the j-th generating unit.

C. Constants

ν	Iteration counter for the iterative process.
N_B	Number of blocks of the cost function for every unit.
N_D	Number of demands in the system.
N_H	Number of price-blocks for every demand.
N_J	Number of units in the system.
N_N	Total number of nodes in the system.
N_T	Number of time periods considered.
$\nu^{\max}$	Maximum number of iterations for the iterative process.
λ_t^M	Market-clearing price corresponding to hour t. Note that λ_t^M is not strictly a constant, its value is derived from the solution of problem (6)–(13).
λ_{dth}^D	Price corresponding to the h-th block of demand d in hour t.
λ_{jtb}^G	Price corresponding to the b-th block of unit j in hour t.
$p_{dth}^{D\,\mathrm{bid}}$	Size of the h-th quantity block offered by demand d in hour t.
$p_{jtb}^{G\,\mathrm{bid}}$	Size of the b-th quantity block offered by unit j in hour t.
Π_j	Feasible operating region for unit j.
B_{nk}	Susceptance of the line between nodes n and k.
G_{nk}	Conductance of the line between nodes n and k.

D. Variables

p_{jt}^G	Power produced by unit j in hour t.
p_{jtb}^G	Pwer produced with the b-th block of unit j in hour t.
p_{dt}^D	Power consumed by demand d in hour t.
p_{dth}^D	Power consumed by the h-th block of demand d in hour t.
q_t^i	Quota of GENCO i in hour t.
$\bar{p}_{nk}^F$	Maximum capacity of the line between nodes n and k.
$p_{nt}^{i,\mathrm{others}}$	Summation of power injected in node n in hour t by all the participants except GENCO i.
θ_{nt}	Phase angle of node n in hour t.

E. Sets

Γ_i	Set of generating units belonging to GENCO i.
Ψ_n	Set of units connected to node n.
Φ_n	Set of nodes directly connected to node n.
Δ_n	Set of demands connected to node n.

I. INTRODUCTION

QUITE a few functioning pool-based electricity markets present oligopolistic structure. That is, they include generating companies (GENCOs) with such structure that they may alter market-clearing prices to their own benefits. These GENCOs are referred to as price-maker GENCOs. The analysis of the functioning of the market if one or several

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GENCOs have market power is complex, particularly in a day-ahead market setting. This complexity arises from the rigorous modeling of the effect of the actual power productions of price-maker GENCOs on market-clearing prices, in conjunction with binary variables, required by the multi-period market setting. However, this analysis is of fundamental importance because it provides the regulator with relevant information to identify and mitigate the exercise of market power. It also provides GENCOS with the appropriate information to maximize their respective profits, within the regulatory framework, by altering market clearing prices to their own respective benefits. The purpose of this paper is to provide a tool that allows the simulation of a pool based electricity market that includes price-maker GENCOs in a day-ahead pool-based market setting. A detailed modeling of the profit maximization behavior of every GENCO is provided. A very detailed market clearing auction that seeks social welfare maximization is embedded in the simulator. Simulation results allow identifying equilibrium patterns. This is further detailed below.

This paper considers a pool-based multiperiod electric energy market [1]–[4]. The market functioning is simulated repeatedly to identify equilibrium patterns. The considered multiperiod framework requires using both continuous and binary variables. Market equilibria analysis is usually addressed in a single period framework and restricted by the assumptions of continuity and convexity [5]–[9]. This is not the case for the procedure presented in this paper that uses an iterative simulation procedure to escape the limitations of imposing continuity and convexity.

In pool-based electricity markets, producers can be either price-takers or price-makers. However, the market is characterized by the dominant behavior of price-maker producers that intend to modify market-clearing prices for their respective benefits. Demands are considered price-sensitive and the elasticity of each load is described through a piecewise constant price-demand curve.

A network-constrained multiperiod auction is used to clear the market [10]. Producers submit price/MWh production bids for each hour and owned unit. They may declare operation constraints for their respective units, but they are not required to do so. Demands submit hourly price/MWh consumption bids in the format of piecewise constant price-demand curves describing their responsiveness to price. The auction-based market-clearing algorithm results in hourly market-clearing prices to be paid by each demand being supplied and to be paid to each unit scheduled to produce.

This day-ahead market functioning is repeatedly simulated. Public information made available by the market operator includes hourly aggregated offer curves and hourly aggregated demand curves. If the transmission network is being modeled, the market operator also provides hourly power injections in each bus. The public information provided by the market operator allows producers to bid in the market pursuing independently the maximization of their respective profits. In turn, the market-clearing algorithm is run resulting in updated public information to be broadcasted by the market operator to market participants. This procedure is repeated a sufficiently large number of times to allow identifying equilibria [5].

The simulation tool presented in this paper complements the continuity- and convexity-based single-period equilibrium analysis proposed by Hobbs *et al.* [5]–[7], and by Smeers *et al.* [8], [9]. On the other hand, it extends the work reported in [11] through an iterative simulation procedure.

The contributions of this paper are the following ones:

1) A detailed simulator of a pool-based multiperiod electricity market is developed.
2) A detailed optimization algorithm to determine the optimal response of a GENCO to the market, taking into account its influence on market clearing prices, is embedded in the simulator.
3) A detailed optimization algorithm to clear the market maximizing social welfare is embedded in the simulator. This algorithm reflects precisely the operating constraints of the individual generators, including minimum up-time, minimum down-time and ramping limits.
4) The effect of price-maker GENCOs on market clearing prices is properly modeled by using price-quota curves.

The simulation tool presented in this paper can be used by the market regulator (i) to monitor market functioning; (ii) to detect the exercise of market power by producers and consumers, and (iii) to identify their respective most profitable equilibria.

Finally, it should be noted that the purpose of the paper is to analyze the market power originated from retaining power (altering bidding prices). Market power related to purposely creating congestion is outside the scope of this paper.

The remainder of this paper is organized as follows. Section II provides models for producers and consumers, describes the auction-based market-clearing algorithm and presents the assumptions made to model the bidding strategy of the GENCOs. Section III describes the market simulation procedure in detail. Section IV presents data, results and conclusions from a realistic case study. In Section V, the main conclusions are summarized. An Appendix provides details on the linearization of losses.

II. Formulation of The Models

A. Generating Companies

Depending on its relative size and generating mix, a GENCO behaves either as a price-maker [11] or as a price-taker; in other words, either the company is able to influence market prices to its own profit, or not. In the latter case, the GENCO takes whichever prices resulting from the market-clearing mechanism. From the modeling point of view, there is no need to explicitly state the price-maker/price-taker difference, because a price-taker can be regarded as a price-maker which price-quota curve is defined by a single step. Hence, the formulation presented next is valid for both price-makers and price-takers.

The formulation of the problem faced by GENCO i is as follows [11]:

$$\underset{p_{jt}^{G}, q_t^i}{\text{maximize}} \quad \sum_{t=1}^{N_T} \left[q_t^i \lambda_t^i \left(q_t^i\right) - \sum_{j \in \Gamma_i} c_{jt} \right] \tag{1}$$

$$\text{subject to:} \quad p_{jt}^{G} \in \Pi_j; \quad \forall j \in \Gamma_i; \quad t = 1, \dots, N_T \tag{2}$$

$$q_t^i = \sum_{j\in\Gamma_i} p_{jt}^G; \quad t=1,\ldots,N_T \tag{3}$$

$$\sum_{j\in(\Psi_n\cap\Gamma_i)} p_{jt}^G + p_{nt}^{i,\text{others}} + \sum_{k\in\Phi_n} B_{nk}(\theta_{nt}-\theta_{kt}) - \frac{1}{2}\sum_{k\in\Phi_n} G_{nk}(\theta_{nt}-\theta_{kt})^2 = 0; \quad t=1,\ldots,N_T; \quad n=1,\ldots,N_N \tag{4}$$

$$-\bar{p}_{nk}^F \le B_{nk}(\theta_{nt}-\theta_{kt}) \le \bar{p}_{nk}^F; \quad n=1,\ldots,N_N; \quad \forall k\in\Phi_n; \quad t=1,\ldots,N_T \tag{5}$$

The objective function (1) expresses the profit of the price-maker: total revenue minus total costs. Taking advantage of the stepwise nature of price-quota curves, the total revenue can be linearly expressed using positive real variables and binary variables. For a detailed formulation of the cost function see [11], [12].

The set of constraints (2) enforces every unit to work within its feasible operating region over the whole planning horizon. A precise mixed-integer linear description of this feasibility region can be found in [12], [13].

The set of constraints (3) expresses the price-maker quota for every hour as the sum of the power production of its units.

The block of (4) defines power balance at every node, stating that the difference between the power reaching any node and the power leaving that node must be equal to zero. The first term in this equation expresses the power injected by the GENCO. The second term, $p_{nt}^{i,\text{others}}$, expresses the total power injected by other participants in the node; it comprises the power injected by other generators minus the power demanded at the node (variable). Note that $p_{nt}^{i,\text{others}}$ is considered publicly available data known by the GENCO before solving its optimization problem. The third term is the net power reaching the node through adjacent lines. Note that 50% of the losses incurred in each of the lines connected to the considered node is introduced as an artificial demand in that node. This mechanism allows formulating a simple yet accurate linear model for the losses. For more details regarding loss modeling see the Appendix and [10].

The block of (5) imposes the restrictions related to the capacity of the transmission lines.

For a given hour, the quota of a price-maker is the amount of power it contributes to serve the demand in that hour. The function that expresses how the market-clearing price changes as the quota of a given price-maker changes is called price-quota curve [11]. Observe that different price-makers competing in the same electricity market present different price-quota curves.

The price-quota curve corresponding to a price-maker for a given hour is a stepwise monotonically decreasing curve that expresses the actual market-clearing price in that hour as a function of its market quota. Price-quota curves are stepwise because (producer/consumer) bids are assumed to be blocks of power at given prices.

The 24 hourly day-ahead price-quota curves provide all the market information a given price-maker needs to self-schedule optimally, i.e., to maximize its benefits. That is, these curves embody the effects of all interactions with competitors and the market functioning rules. Once these curves are available, the self-scheduling problem of a price-maker can be formulated independently of the problems of other producers.

The day-ahead price-quota curves of a price-maker can be obtained (i) by market simulation or (ii) using forecasting procedures; however both techniques are outside the scope of this paper. For the case studies presented in this paper, a direct method is used to obtain the price-quota curves. For any given hour, the price-quota curve for a certain GENCO is equal to the aggregated demand curve minus the aggregated offer curve of the rest of the GENCOs. Note that the above two curves are assumed to be publicly available from the market operator. For the sake of illustration, Fig. 1 shows a typical price-quota curve.

B. Consuming Companies

Consuming companies (CONCO's) are modeled in a simple fashion because the main purpose of the procedure presented in this paper is to analyze the behavior of GENCOs. Each demand is considered a fixed set of price-quantity values, i.e., consumption offers are the same at every iteration. See Fig. 2 below for an example of a generic demand function. As in the real world, stepwise elastic demands are considered distributed over the nodes of the network.

In the proposed model, every CONCO offers to consume a maximum of 4 blocks of demand for every hour.

C. Market-Clearing Algorithm

A network-constrained multiperiod auction to maximize social welfare is used to clear the market. It is based on mixed-integer linear programming The complete formulation of the problem is as follows:

$$\underset{p_{jtb}^G,\,p_{dth}^D}{\text{maximize}} \quad \sum_{t=1}^{N_T}\left[\sum_{d=1}^{N_D}\sum_{h=1}^{N_H} p_{dth}^D\lambda_{dth}^{D\,\text{bid}} - \sum_{j=1}^{N_J}\sum_{b=1}^{N_B} p_{jtb}^G\lambda_{jtb}^{G\,\text{bid}}\right] \tag{6}$$

$$p_{jt}^G \in \Pi_j; \quad j=1,\ldots,N_J; \quad t=1,\ldots,N_T \tag{7}$$

$$0 \le p_{dth}^D \le p_{dth}^{D\,\text{bid}} \quad d=1,\ldots,N_D; \quad h=1,\ldots,N_H; \quad t=1,\ldots,N_T \tag{8}$$

$$0 \le p_{jtb}^G \le p_{jtb}^{G\,\text{bid}} \quad j=1,\ldots,N_J; \quad b=1,\ldots,N_B; \quad t=1,\ldots,N_T \tag{9}$$

$$\sum_{b=1}^{N_B} p_{jtb}^G = p_{jt}^G; \quad j=1,\ldots,N_J; \quad t=1,\ldots,N_T \tag{10}$$

$$\sum_{h=1}^{N_H} p_{dth}^D = p_{dt}^D; \quad d=1,\ldots,N_D; \quad t=1,\ldots,N_T \tag{11}$$

$$\sum_{j\in\Psi_n} p_{jt}^G + \sum_{k\in\Phi_n} B_{nk}(\theta_{nt}-\theta_{kt}) = \sum_{d\in\Delta_n} p_{dt}^D + \frac{1}{2}\sum_{k\in\Phi_n} G_{nk}(\theta_{nt}-\theta_{kt})^2; \quad t=1,\ldots,N_T; \quad n=1,\ldots,N_N \tag{12}$$

$$-\bar{p}_{nk}^F \le B_{nk}(\theta_{nt}-\theta_{kt}) \le \bar{p}_{nk}^F; \quad n=1,\ldots,N_N; \quad \forall k\in\Phi_n; \quad t=1,\ldots,N_T \tag{13}$$

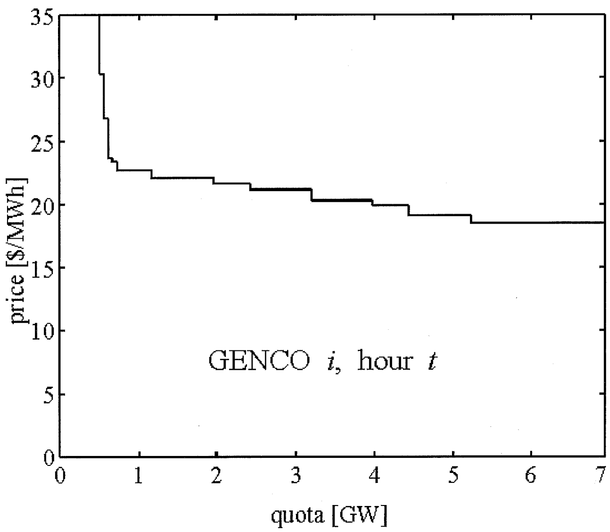

Fig. 1. Price-quota curve.

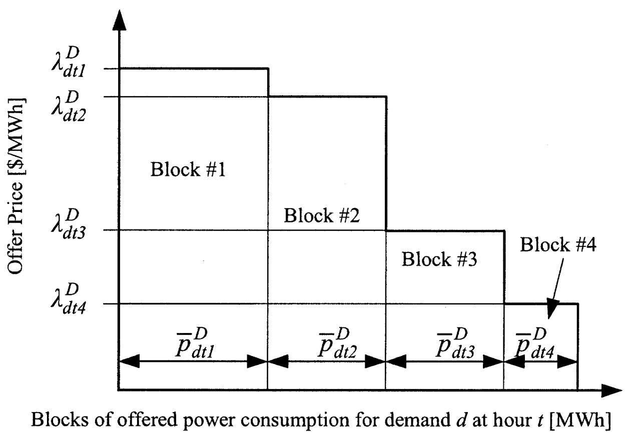

Fig. 2. Example of stepwise elastic demand.

Equation (6) is the objective function; it expresses total social welfare as the summation of the social welfare for every hour. Social welfare is computed as the difference of two terms: the first term is the sum of accepted demand bids times their corresponding bidding prices; the second term is the sum of accepted production bids times their corresponding bidding prices. It should be noted that if the producers do not bid at their marginal costs the difference of the two terms is not the actual social welfare, but the "declared" social welfare.

The block of (7) is equivalent to block (2) but extended to all the units in the system. The blocks of (8)–(9) state the limits for the main variables of the problem. The block of (10) defines the power generated by any generator in any given hour as the summation of its corresponding production blocks. The block of (11) defines the power consumed by any demand in any given hour as the summation of its corresponding consumption blocks. The block of constraints (12) defines power balance at every node, stating that the total generation at any node plus the net injections through lines must equal the total power demanded at that node. Note that artificial demands have been introduced to take losses into account (see the Appendix). The block of constraints (13) is equivalent to block (5).

Once problem (6)–(13) is solved, the market clearing price for each hour is obtained as the price of the most expensive production bid that has been accepted in that hour. Note that problem (6)–(13) includes binary variables and therefore its shadow prices are not mathematically defined.

D. Bidding Strategy

The solution of problem (1)–(5) provides any GENCO with its optimal self-scheduling, i.e., the power blocks the GENCO should get accepted in the market to maximize its profit. To that end, its bidding strategy for any given hour is defined as follows:

1) All power blocks with optimal self-scheduling values different from zero are offered at their corresponding marginal costs.
2) The remaining blocks are offered at price infinity.

It is considered that all GENCOs follow the above bidding rule. Note that other consistent sets of assumptions can be made on GENCO behavior. However, the assumptions made are simple and economically consistent.

Although GENCOs do take into account the network to compute their bids, they do not try to use the network as an instrument to exert market power; in other words, no GENCO is trying to produce a congestion in order to take advantage of the resulting higher prices. The modeling of such a behavior is complicated and is outside the scope of this paper.

III. Market Simulation

The simulator described in this paper considers the three typical participants in a pool-based electricity market, namely, generating companies (GENCOs), consuming companies (CONCO's) and the market operator (MO). This section describes the iterative process used for the simulations. The main steps of this process are described below:

STEP 0. An initial solution and initial price-quota curves are obtained for all GENCOs by clearing a market considering that all units offer all their power blocks for all time periods at their corresponding marginal costs. This provides an initial solution. Prices derived from this solution are lower bounds for final prices and productions are upper bounds for final productions.

STEP 1. Once the market is cleared, all necessary information is made available to the participants. The aggregated offer and the aggregated demand are made public for every hour. Injections at all nodes for every hour are also publicly available.

STEP 2. With the information obtained from STEP 1 and with the knowledge of its own previous offer to the market, every GENCO derives its price-quota curve for every hour [1]. Assuming that all other companies do not change their offers, any given GENCO solves problem (1)–(5) described above. The solution obtained allows the GENCO to derive its optimal offer for the next iteration.

STEP 3. Once all GENCOs have calculated and submitted their offers, the MO clears the market and calculates productions and market-clearing prices for every hour; this is achieved solving problem (6)–(13) above. If the desired number of iterations has been reached the simulation concludes; otherwise, the simulation continues in STEP 1.

The purpose of the simulation is to identify alternative market equilibria considering that all GENCOs try to maximize their respective profits modifying market-clearing prices. To do so, the market functioning is simulated a sufficient number of times without any convergence criterion. Therefore, no termination criterion other than "simulate a sufficient number of times" is needed.

A flow-chart describing the algorithm above is shown in Fig. 3:

Note finally that the purpose of the above procedure is to determine the strategy of every GENCO and to optimally 'combine' all those strategies through the market clearing algorithm to simulate the market results. The whole procedure is repeated a sufficient number of times to identify patters in market results.

IV. Case Study

The simulator has been tested using an all-thermal power system of realistic size. The considered market includes 3 price-maker GENCOs owning 19, 13, and 11 units, respectively (see Table I), and 7 single-unit price-taker GENCOs. The total number of units is therefore 50. In the market clearing procedure, all unit constraints, including minimum up-time, minimum down-time and ramping limits, are considered. The market time horizon is 24 hours. Data regarding the ownership of the units among the GENCOs is presented in Table I.

Data for all units are based on the 1996 IEEE RTS [14], and are detailed in Table II. In this table, 'Type' indicates the unit type (A, B, C, D, E, F or G); $\bar{\mathrm{P}}$ and $\underline{\mathrm{P}}$ indicate, respectively, maximum and minimum power output; every '$\mathrm{C_b}$' value provides the production cost of the block 'b' of the unit (four-block piecewise convex cost curves are considered); 'RR' gives both ramp-up and ramp-down maximum values; 'SC' is the constant start-up cost; and 'MUT' and 'MDT' represent the minimum up- and down-times, respectively.

Fig. 4 depicts the distribution of the total generating capacity in the system among the participating GENCOs. For the sake of clarity, the power of the 7 small price-taker GENCOs is plotted as one single piece in the pie chart.

The units are distributed over a transmission network including 73 nodes and 108 lines, which is also based on the network described in [14]. The units are placed evenly distributed over the network, avoiding the possibility of local market power taking place in some areas. The system analyzed is therefore reasonably realistic.

Three different case studies are presented in this section. The first one is the 'Base Case,' in which the previously described market is simulated; the second case study differs from the 'Base Case' only in the fact that the transmission network is not considered; the third and last case study is different from the 'Base Case' in the fact that no market power exertion is allowed. This is achieved by imposing an independent operation to every unit regardless of unit ownership. The network is considered in this

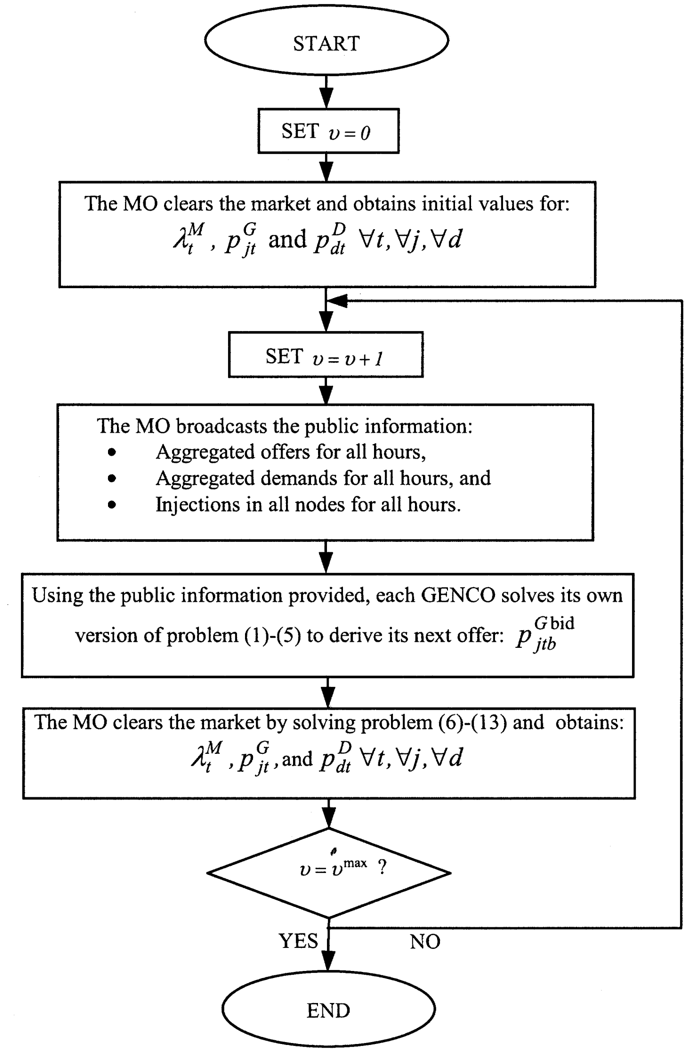

Fig. 3. Flowchart describing the proposed iterative algorithm.

TABLE I
UNITS OWNED BY EACH GENCO

	GENCO									
	1	2	3	4	5	6	7	8	9	10
Type A	2	1	2	-	-	-	-	-	-	-
Type B	2	3	2	-	-	-	-	-	-	-
Type C	2	3	2	-	-	-	-	-	-	-
Type D	2	1	1	1	1	-	-	-	-	-
Type E	2	3	2	-	-	1	1	-	-	-
Type F	3	1	1	-	-	-	-	1	1	-
Type G	6	1	1	-	-	-	-	-	-	1
Total	19	13	11	1	1	1	1	1	1	1

third case study. For each case study 30, iterations of the simulation process are performed. Results in terms of prices and productions are analyzed below.

A. Case 1: Base Case

In the 'Base Case,' the market previously described is simulated. Fig. 5 shows the evolution of the total power accepted in the market for production in three representative hours. For

TABLE II
GENERATING UNITS DATA

Type	A	B	C	D	E	F	G
$\overline{P}$ [MW]	12	76	100	155	197	350	400
$\underline{P}$ [MW]	2.4	15.2	25	54.25	68.95	140	100
$C_1^{(*)}$	25.63	18.98	20.36	10.95	21.02	11.13	7.82
$C_2^{(*)}$	26.01	19.81	21.92	11.32	22.24	11.79	7.92
$C_3^{(*)}$	29.38	23.01	23.72	11.79	23.22	12.25	8.14
$C_4^{(*)}$	33.28	26.46	24.87	12.43	24.22	12.94	8.34
RR[MW/h]	12	76	100	155	180	120	400
SC($)	114.1	789.6	949.9	1263	1300	5920	NA$^{(**)}$
MUT (h)	4	8	8	8	12	24	NA$^{(**)}$
MDT (h)	2	4	8	8	10	48	NA$^{(**)}$

$^{(*)}$Units: [$/MWh]. $^{(**)}$ NA: Not Applicable.

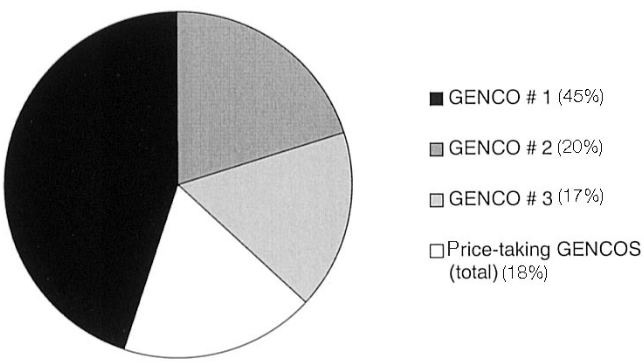

Fig. 4. Distribution of the total generating capacity among the GENCOs.

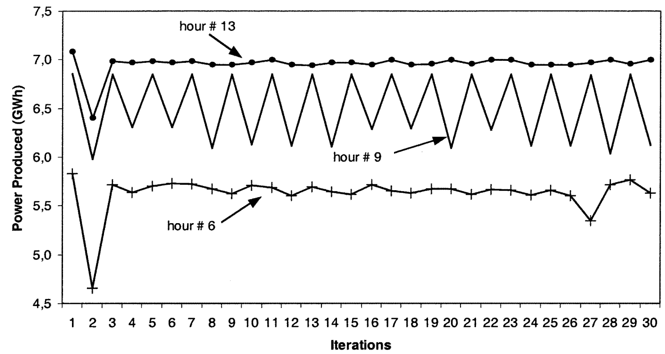

Fig. 5. Total production for three different hours. Base Case.

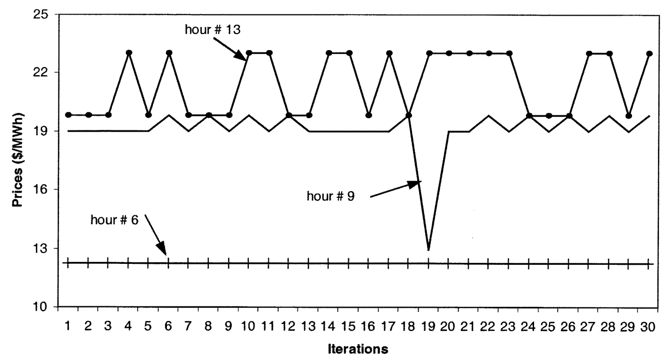

Fig. 6. Market-clearing prices for three different hours. Base Case.

some hours, the total production is almost stable after iteration number 4. This can be interpreted as an indication of high competition, (high offer and low demand). This is the case of hours 6 and 13. However, for other hours, as for instance hour 9, a two-level cycle is apparent. In hour 9 production is high for odd iterations and low for even iterations. This effect can be interpreted considering that lower prices in an iteration result in smaller offered quantities in the next iteration and vice versa. For hours like hour 9, the exercise of market power is apparent and no stable solution can be found. See [15], [16] for more examples of this behavior in a different setting.

Fig. 6 shows the evolution of prices for the same hours as Fig. 5. For some hours prices are stable during the iterations, e.g., hour 6. As previously stated, this means that for those hours competition is high. For other hours, a nonstable behavior is apparent. That is, for some iterations the price is high and for others it is low. Although prices are not stable, they are confined to a small set of different prices.

Note that for hour 13, very small production differences (see Fig. 5) are related to significant price changes (see Fig. 6). The reason is that the slope of the aggregated offer curve is high around those production values for that hour.

Final productions and demands result in a slightly congested network, i.e., 2 lines reach their capacity limits.

Throughout the iterations, the inter-temporal constraints of some units become active, specially ramping limits for units of type G. The reason is that type-G units are cheap and their ramping constraints are relatively strict. Additionally, some minimum down-time constraints become active and force some units not to start until these constraints are satisfied. Minimum down-time constraints are particularly binding for type-B units. Nevertheless, the influence of these technical constraints and others in the evolution of prices and productions is small and their enforcement does not change the general behavior of GENCOs, i.e., GENCOs have to enforce them but their strategies are not strongly influenced by these technical constraints.

B. Case 2: No Network

In this case, the same analysis performed in the 'Base Case' is carried out, but without taking the network into account. This means that all line capacities are large enough to avoid congestion and that all lines are assumed to be lossless. Making these assumptions, results for this case study do differ from the results for the 'Base Case' in some aspects. Productions do not change significantly (compare Fig. 7 with Fig. 5). However, prices are lower as shown in Fig. 8 (compare with Fig. 6). The reason for that is that the absence of network congestion makes cheaper generation available and that results in lower prices.

Furthermore, in the 'No Network Case,' price oscillation becomes higher because damping network effects are not present.

C. Case 3: No Market Power

The third case differs from the previous two cases in the fact that all units are forced to work as if they were independent units; i.e., every unit is considered to be a GENCO. Therefore, the possibility for market power exertion is dramatically

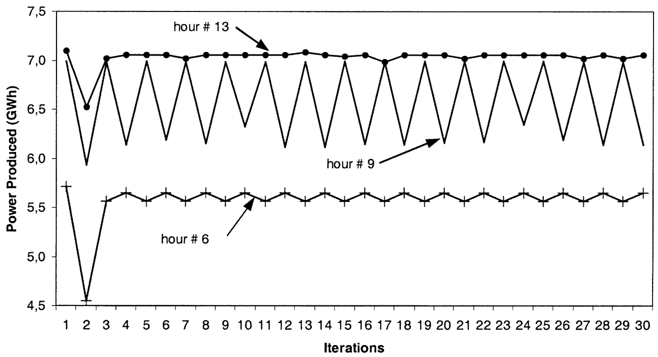

Fig. 7. Total production for three different hours. No Network Case.

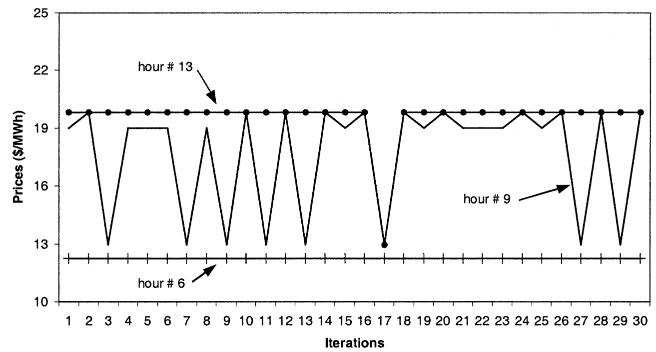

Fig. 8. Market-clearing prices for three different hours. No Network Case.

reduced. In fact, each of the largest units (type G units) represent less than 5% of the total installed capacity. The network is considered in this case.

The elimination of the possibility of market power exertion clearly affects the results of the simulation. As shown in Fig. 9, production oscillations are very small for all hours. In the absence of market power, incentives to withhold production are inexistent and, hence, generators tend to bid their whole capacities.

Note that the main difference between Figs. 5 and 9 is the production evolution in hour 9. As previously stated, the oscillating behavior in hour 9 for the 'Base Case' (see Fig. 5) is due to market power exertion by price-maker GENCOs. However, when market power exertion is not allowed, the oscillation disappears (as it is observed comparing Figs. 5 and 9 for hour 9). Moreover, productions in hours 6 and 13 are obtained under high competition in the 'Base Case'; hence, they are not very influenced by the elimination of market power, as can be seen comparing Figs. 5 and 9 for both hour 6 and 13.

Fig. 10 shows the stable evolution of prices for this case. After two iterations prices become constant. This is a definitive indication that market power is not being exerted. Note that prices are not only fixed, but they are really small compared to the 'Base Case.'

Network congestion in this case is similar to network congestion in the 'Base Case.'

D. Comparison Among Profits Made by the Largest GENCO in the Three Case Studies

In this section, a comparison is made among the profits made by the largest GENCO in the three case studies. Fig. 11 shows the evolution of the total profit made by GENCO #1 in the first two case studies.

For the 'No Market Power Case,' the sum of the profits made by the first 19 units is plotted because those are the units that belong to GENCO #1 in the other cases.

Fig. 11 shows that profits for both the 'Base Case' and the 'No Network Case' have an oscillating behavior because prices and quantities are not stable. Nevertheless, actual profit values are similar in both cases. In the 'No Network Case,' the fact that no network is considered does not have a relevant effect on profits, although it does affect prices. The explanation for that is that while revenue decreases due to lower prices, costs also decrease because the cheapest units are always used.

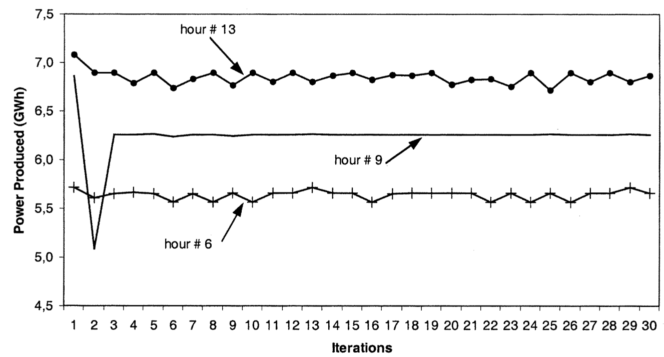

Fig. 9. Total production for three different hours. No Market Power Case.

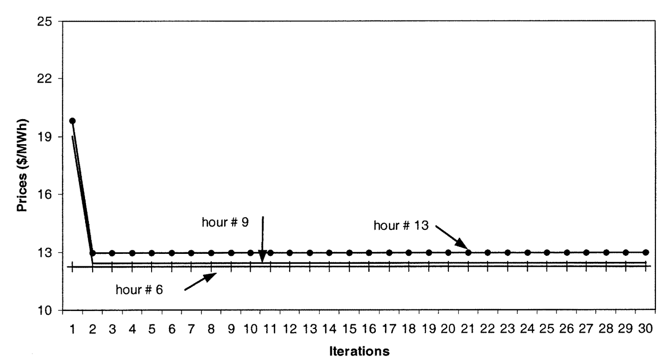

Fig. 10. Market-clearing prices for three hours. No Market Power Case.

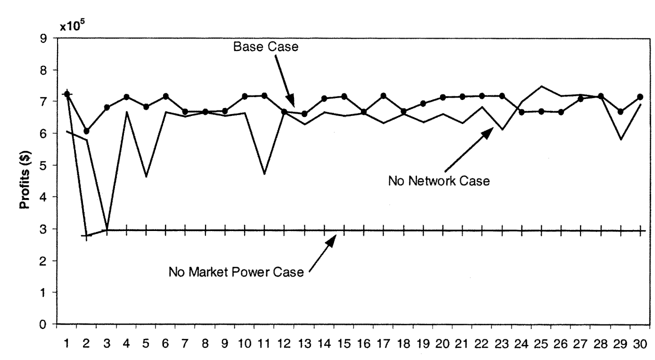

Fig. 11. Profits made by the first GENCO in the three cases.

Regarding the 'No Market Power Case,' Fig. 11 shows that profits are constant. Moreover, profits are substantially lower than in the previous cases because the market is more competitive in this case. Note that stability comes from the fact that no company can profit from withholding power, hence its whole capacity is always offered.

The simulations presented are performed using CPLEX 7.5 under GAMS [17] on a SGI R12000 (400 MHz) processor with 500 MB of RAM. The total required CPU times are 1055.8 minutes for the 'Base Case,' 84.1 minutes for the 'No Network Case' and 3681.8 minutes for the 'No Market Power Case.' This represents an average of 35.9, 2.8, and 122.7 minutes per iteration, respectively. The total number of mixed-integer linear programming problems that have to be solved is 330, 330, and 1530, respectively. The average minutes needed per problem are 3.20, 0.25, and 2.41, respectively. Note that computing times decrease by a factor near 10 when the network is not considered. Average CPU time to solve the optimization problem of the 19-unit GENCO is 3.552 minutes for Case 1 and 0.173 minutes for Case 2. Average CPU time to solve the optimization problem of one single-unit GENCO is 2.927 minutes for Case 1, 0.112 minutes for Case 2 and 2.424 minutes for Case 3. Average CPU time to solve the market clearing optimization problem is 1.761 minutes for Case 1, 1.044 minutes for Case 2 and 2.792 minutes for Case 3. Note that this analysis is not intended for real-time use.

The major conclusions from the case studies are summarized below:

1) Price-maker GENCOs do exert market power in some hours to increase their respective profits.
2) Price-maker GENCOs that coordinate the production of their units increase highly their respective profits compared to GENCOs which units act independently. In the case studies provided, differences up to 100% occur.
3) The exercise of market power by price-maker GENCOs results in oscillating productions and prices for some hours. And this results in increasing market uncertainty.
4) Depending on demand levels, different equilibrium patterns can be found for different hours within the same market horizon. For example, in the 'Base Case,' hour 6 shows nearly perfect competition, whereas hour 9 shows the effects of market power exertion.
5) If the number of congested lines is not high and the congested lines do not create drastic bottlenecks, GENCO profits do not significantly change if the network is ignored.

V. Conclusion

This paper presents a multiperiod simulation tool to identify market equilibria in a pool-based electric energy market. Market participants are GENCOs, either price-takers or price-makers, consumers and the market operator. The market operator clears the market using a multiperiod network-constrained auction that results in hourly market-clearing prices. Congestion management is implicitly and optimally achieved within the auction algorithm. Network losses are properly taken into account. The bidding and market-clearing procedures are repeatedly simulated using the tool proposed in this paper to identify equilibria in the market. The simulation tool is flexible and adaptable to different market settings. Extensive computational simulations using different electric energy systems of realistic size show the formation of complex equilibria with identifiable patterns throughout the hours of the considered market horizon. These patterns, related to prices and quantities, are described, compared and analyzed in this paper. The simulation tool presented is particularly appropriate for the market operator to monitor the market and to identify oligopolistic behavior and its associated equilibria. It is also useful for a GENCO to identify, within a regulatory framework, its most profitable behavior and the resulting equilibria.

Appendix
Linear Approximation of Losses

The losses incurred in line n-k are represented as additional fictitious demands in buses n and k, respectively. Therefore, the additional demand associated with node n representing losses is given as

$$p_n^{\text{loss}} = \frac{1}{2} \sum_{k \in \Phi_n} p_{nk}^{\text{loss}} \tag{A.1}$$

Note that subindex t representing time dependence has been dropped for convenience. The term p_{nk}^{loss} in (A.1), is defined as follows (see (4) and (12) in Section II):

$$p_{nk}^{\text{loss}} = G_{nk}(\theta_n - \theta_k)^2 = G_{nk}(\theta_{nk})^2 \tag{A.2}$$

where θ_{nk} is the voltage angle difference between buses n and k.

A linear approximation of losses can be obtained using L piece-wise linear blocks [10].

$$p_{nk}^{\text{loss}} = G_{nk} \sum_{\ell=1}^{L} \alpha_{nk}(\ell)|\theta_{nk}(\ell)| \tag{A.3}$$

where $\theta_{nk}(\ell)$ is the ℓth voltage angle block relative to buses n and $k, \alpha_{nk}(\ell)$ is the slope of the ℓth block of angle and L is the number of blocks of the linearization of losses.

If the piece-wise length is $\Delta\theta$, the slope can be expressed as

$$\alpha_{nk}(\ell) = (2\ell - 1)\Delta\theta \tag{A.4}$$

and the power loss is calculated as

$$p_{nk}^{\text{loss}} = G_{nk}\Delta\theta \sum_{\ell=1}^{L} (2\ell - 1)|\theta_{nk}(\ell)| \tag{A.5}$$

The absolute value function is expressed as

$$|\theta_{nk}(\ell)| = \theta_{nk}^{+}(\ell) + \theta_{nk}^{-}(\ell) \tag{A.6}$$

Finally, we obtain

$$p_n^{\text{loss}} = \frac{1}{2} \sum_{k \in \Phi_n} \left[G_{nk}\Delta\theta \sum_{\ell=1}^{L} (2\ell - 1)(\theta_{nk}^{+}(\ell) + \theta_{nk}^{-}(\ell)) \right] \tag{A.7}$$

which expresses linearly losses as a function of angle variables.

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