Optimal Response of an Oligopolistic Generating Company to a Competitive Pool-Based Electric Power Market

Antonio J. Conejo, *Senior Member, IEEE*, Javier Contreras, *Member, IEEE*, José Manuel Arroyo, *Student Member, IEEE*, and Sebastián de la Torre, *Student Member, IEEE*

***Abstract*—The target of an oligopolistic generating company in a pool-based electric power market is to maximize its profits using two related instruments at hand: 1) its ability to modify the market-clearing price and 2) its capability to alter its own production level. Power balance is not an issue for the generating company; the independent system operator ensures power balance considering generator and demand bids through any market-clearing procedure. This paper proposes a mathematical model to determine the output of the generators owned by an oligopolistic generating company so that its profit is maximized for a one-day to one-week time horizon. An efficient solution technique to solve the resulting large-scale discontinuous nonlinear mixed-integer optimization problem is reported. A case study that illustrates the proposed model and the solution technique developed is analyzed in detail.**

***Index Terms*—Electric power market, market power, oligopolistic generating company, power pool, price–quota function, stepwise nonlinear mixed-integer optimization.**

I. Introduction

QUITE a few of present day pool-based electric power markets present an oligopolistic structure. This is the case of the electric markets of England and Wales [1] and mainland Spain [2]. Through either the Independent System Operator (ISO) or the Power Exchange (PX), the market ensures power balance considering generator and demand bids [3]. The role of a multi-machine generating company (GENCO) is therefore not to ensure power balance at minimum cost but to submit bids to the ISO with the target of maximizing its own benefits.

The motivation of this paper is twofold. First, to provide the Regulator with a tool to measure the market power of an oligopolistic GENCO (O-GENCO). Using that information the Regulator can set up operation rules to prevent an unfair behavior from O-GENCOs. Secondly, to provide the O-GENCO with a tool to maximize its profits, within the regulatory framework, by adequately modeling its market power.

In an electricity market, an O-GENCO has two coupled instruments available to maximize its benefits: 1) its ability to modify the market-clearing price as a result of its market power and 2) its capability to alter its own production level of energy and reserve. The manner in which an O-GENCO should produce to maximize its profits, is a complex dynamic decision problem which is addressed in this paper. For a time horizon varying from one day to one week, this problem can be formulated as a large-scale nonlinear discontinuous (stepwise) mixed-integer optimization problem with exploitable structure. The solution of this problem provides the O-GENCO with sound information to elaborate its bidding strategy. The proposed solution technique is an efficient coordinate-descent technique [4] coupled with mixed-integer linear programming techniques [5] yielding an efficient and accurate solution procedure. The elaboration of the actual bidding strategy taking into account the reactions of competitors is a related problem, which is, however, outside the scope of this paper. Furthermore, no network constraints are considered in this paper. This paper provides the following.

1) A characterization of how the market-clearing price changes with the production level (quota) of the O-GENCO. The market–quota at hour k of an O-GENCO is defined as its total production in that hour.
2) A formulation and characterization of the problem whose solution provides the O-GENCO production strategy to maximize its profits.
3) A computationally efficient solution procedure for the complex problem formulated in 2).
4) The analysis of a realistic case study.

It should be emphasized that the proposed model explicitly recognizes and takes advantage of the discontinuous stepwise dependency of the market-clearing price with the production quota of an O-GENCO. The model developed assumes that every GENCO (oligopolistic or otherwise) has available, through forecasting or simulation, its corresponding hourly price–quota curves, i.e., the functions that relate, for every hour, the market-clearing price (price) and the total production of the GENCO (quota). The model can be used in a market including one or several O-GENCOs, as well as any number of nonoligopolistic GENCOs. If several O-GENCOs compete in a given market, forecasting accuracy of the above curves deteriorates, but the formulation provided remains as stated.

Although the technical literature is rich in references on the modeling of electric power markets [1], [3], [6]–[11], so far, not great attention has been paid to the oligopolistic case in power engineering journals. Besides, most references treating the oligopolistic case do so using standard microeconomic frame-

Manuscript received March 3, 2000; revised July 11, 2001. This work was supported by the Ministerio de Educación y Cultura of Spain and European Union Projects DGICYT PB95-0472, FEDER-CICYT 1FD97-0545, and Grant FPU-AP99.

The authors are with the Departamento de Ingeniería Eléctrica, Electrónica y Automática, Escuela Técnica Superior de Ingenieros Industriales, Universidad de Castilla-La Mancha, Ciudad Real, Spain (e-mail: Antonio.Conejo@uclm.es; Javier.Contreras@uclm.es; JoseManuel.Arroyo@uclm.es; storre@ind-cr.uclm.es).

Publisher Item Identifier S 0885-8950(02)03803-8.

works not particularly adapted to the power industry [12]. In addition, [13] provides an empirical study of the market power in the electricity market of England and Wales. From a power industry perspective, [7] and [14] provide models that simplify the relationship between the market–quota of the O-GENCO and the market-clearing price. Further relevant simplifications are also introduced in order to solve the resulting optimization problem. The modeling and the solution technique proposed in this paper require neither relevant modeling nor computational simplifications.

This paper is organized as follows. Section II provides a detailed formulation of the problem, free of important simplifying assumptions. The price–quota dependence is first analyzed and then the profit maximization problem for an O-GENCO is formulated in detail. In Section III the proposed solution technique is described. In Section IV a case study is analyzed. In Section V some relevant conclusions are drawn. The cost function and the operating constraints of the thermal units are stated in Appendix A. An illustrative example is solved in Appendix B.

II. Formulation

The price–quota dependency is first analyzed and then the profit maximization problem of an O-GENCO is stated.

A. Price–Quota Dependence

In current pool-based electricity markets every generator submits a list of bidding power blocks and their corresponding prices to the ISO for every hour of the planning horizon. The ISO uses either a single- or a multiround market-clearing mechanism. Examples of single-round markets are the ISO of England and Wales [1] and the one of mainland Spain [2]. A formal description of bidding systems and market-clearing mechanisms is provided in [3]. Under general nonrestrictive assumptions, the market-clearing price results in a stepwise monotonically decreasing function of the market–quota of the O-GENCO. If several O-GENCOs participate in the market the above statement remains true. The discontinuous nature of this curve is the result of using either single- or multiple-block bids. The decreasing behavior is a consequence of the different costs of the generating units of the O-GENCO. If the O-GENCO offers no power for a given hour the resulting market-clearing price is high because the cheap units of the O-GENCO are not allocated to produce. On the other hand, if the O-GENCO offers all its available units at prices close to their corresponding marginal costs, the market-clearing price decreases as a result of the cheap units provided by the O-GENCO and allocated to produce. In between these two extreme possibilities, the market-clearing price for any given hour is a stepwise monotonically decreasing function of the power-quota of the O-GENCO. It should be noted that the above price–quota behavior also occurs for all GENCOs, but it is particularly relevant for O-GENCOs.

The price–quota curves can be determined by using information of previous competitors' bidding behavior. In the electricity market of mainland Spain, aggregate bidding (offer and demand) curves are available on-line (http://www.omel.es); in the Californian market (before the 2001-crack), this information was available with a three-month delay. Alternatively, a market simulator can be used to estimate these curves. For the remaining of this paper price–quota curves are assumed known. These price–quota curves are also called residual demand curves [3].

The actual price–quota curve faced by an individual market participant is equal to the consumer demand function minus the sum of all the price-quantity bid functions offered by the rest of the competitors. Following the dynamic Cournot equilibrium concept formulated by Borenstein *et al.* [15], the market agents can be classified as follows.

- *Price Takers*: Their offer is not needed to match the demand. Slight variations in their bids result in slight changes in the market price. They are also called "competitive fringe."
- *O-GENCOs*: Without them the demand cannot be covered. They have a big impact over the price due to their market power.

In a real market both types of agents would compete against each other, and their respective bidding strategies would be quite different. Since the market demand can be matched without any given price taker, its optimal price/quantity bid would be in a certain spot of its corresponding price–quota curve. It should also be noted that a price taker has no power to significantly alter the market-clearing price. If an O-GENCO decides not to produce, the market price becomes the unserved energy price, since its presence is essential to cover the market demand. Furthermore, the O-GENCO has the ability to alter the market-clearing price by fixing its own production level, i.e., its spot in the price–quota curve. An O-GENCO typical price–quota curve is provided in the case study (Fig. 2).

B. Profit Maximization Problem

The profit maximization problem of an O-GENCO is formulated as the following.

$$\text{maximize}_{\lambda,\, q,\, p_i,\quad \forall i} \sum_{k \in K} \left[\lambda_k \left(q(k) \right) q(k) - \sum_{i \in \Omega} c_i(k) \right] \tag{1}$$

subject to

$$\sum_{i \in \Omega} p_i(k) = q(k), \qquad \forall k \in K \tag{2}$$

$$p_i(k) \in \Pi_i \qquad \forall i \in \Omega, \quad \forall k \in K \tag{3}$$

where i is the power unit index; k is the time period index; $c_i(k)$ is the total production cost of unit i in period k; $p_i(k)$ is the power produced by unit i in period k [p_i is the vector of all $p_i(k)$s]; $q(k)$ is the market–quota (power production) of the O-GENCO in period k [q is the vector of all $q(k)$s]; $\lambda_k(q(k))$ is the market-clearing price for a level of production (quota) $q(k)$ by the O-GENCO in period k [λ is the vector of all $\lambda_k(q(k))$s]; K is the set of time periods; Ω is the set of power units owned by the O-GENCO, and Π_i is the set of production constraints of power unit i.

It should be noted that the optimization variables are $\lambda_k(q(k))$s, $p_i(k)$s and $q(k)$s. Note, however, that variables $q(k)$s are straightforwardly obtained from variables $p_i(k)$s.

The objective function (1) expresses the profit of the O-GENCO, which is equal to its revenue minus its production cost. Its revenue is equal to the market-clearing price (which depends on the O-GENCO production–quota) times its power production (market–quota). Its cost function is described in Appendix A. It should be noted that this objective function is discontinuous and nonlinear. It could be modified to include reserve related revenues in case that simultaneous energy and reserve electric power markets co-exist.

The block of constraints (2) simply expresses, for every time period, that the sum of the production of every unit belonging to the O-GENCO is equal to the power output of the O-GENCO as a whole (quota).

The block of constraints (3) expresses in a compact way the operating constraints, for every time period, of every unit belonging to the O-GENCO, i.e., minimum up and down times, ramp rate limits and generation limits. These constraints are further described in Appendix A.

The above large-scale optimization problem cannot be solved by direct application of standard optimization software. This is a consequence of its discontinuous, nonlinear, and large-scale nature.

III. Solution Technique

Problem (1)–(3) is large-scale because the number of units of the O-GENCO and the number of time periods of the planning horizon can both be high. For instance, thirty units and a time horizon of one day hour by hour result in a problem with thousands of variables and constraints. The above problem is nonlinear because the production cost is a nonlinear function of the power production and because of the product of variables λ and q. It is mixed-integer because the modeling of the start-up and shut-down of the units requires the use of binary variables (see Appendix A). In addition, it is discontinuous because the market-clearing price is a stepwise function of the O-GENCO total production.

It should be emphasized that the market-clearing price is a stepwise function of the O-GENCO market–quota (its total production). This fact is reflected in the notation for prices where $\lambda_{k,\,l}$ indicates the price corresponding to the lth step of the O-GENCO price–quota function in hour k (see Fig. 2 in the case study).

It should be noted that the number of steps of the market-clearing price is small for reasonably small changes in the market–quota, e.g., a variation of 20% in the market–quota typically results in no more than 10 price–quota steps. As a consequence of this small number of steps, a coordinate descent solution algorithm [4] coupled with mixed-integer linear programming [5] is suitable to solve problem (1)–(3).

In the algorithm below, it should be noted that subscript k denotes the considered time periods, i.e., the 24 hours of the day. However, dynamic subscript j is used in the algorithm in order to consider time periods one at a time.

The proposed solution technique works as follows.

Step 0) An initial market-clearing price is fixed for every hour $\lambda_{k,\,\hat{l}} \leftarrow \lambda_{k,\,l}^{(0)}\ \forall\, k$. The time period counter is initialized, i.e., $j = 1$. In any iteration, all time periods are considered one at a time. The iteration counter is also initialized, i.e., $IT = 1$.

Step 1) For every possible price $\lambda_{j,\,l}$ of the market-clearing price at time period j the problem below (4)–(8) is solved. Therefore, if the price–quota function for hour j has N_j different steps, N_j problems need to be solved in this step of the algorithm

maximize$_{q,\, p_i,\, \forall\, i}$
$$\sum_{\substack{k\neq j\\ k\in K}} \left(\lambda_{k,\,\hat{l}}q(k)\right) + \lambda_{j,\,l}q(j) - \sum_{k\in K}\sum_{i\in\Omega} c_i(k) \tag{4}$$
subject to
$$\sum_{i\in\Omega} p_i(k) = q(k) \qquad \forall\, k\in K \tag{5}$$
$$p_i(k)\in \Pi_i \qquad \forall\, i\in\Omega,\quad \forall\, k\in K \tag{6}$$
$$\underline{q}_{\hat{l}}(k) \le q(k) \le \overline{q}_{\hat{l}}(k) \qquad \forall\, k\neq j,\, k\in K \tag{7}$$
$$\underline{q}_{l}(j) \le q(j) \le \overline{q}_{l}(j) \tag{8}$$

where $\lambda_{k,\,\hat{l}}$s are fixed market-clearing prices, and $\underline{q}_{\hat{l}}$ and $\overline{q}_{\hat{l}}$ are respectively the lower and upper bounds of the market–quota for those prices.

Step 2) The step price corresponding to the problem whose solution produces maximum profit in Step 1 is determined, $\lambda_{j,\,l^*}$; the corresponding objective function value is P^*. Market-clearing price in hour j is updated as $\lambda_{j,\,\hat{l}} \leftarrow \lambda_{j,\,l^*}$. Therefore, only the l^*th price of the price–quota curve for hour j is used until hour j is considered again. Market–quota bounds are also updated: $\underline{q}_{\hat{l}}(j) \leftarrow \underline{q}_{l^*}(j)$, $\overline{q}_{\hat{l}}(j) \leftarrow \overline{q}_{l^*}(j)$.

Step 3) Being T the number of hours of the time horizon, if the objective function has not improved during the last T time periods, then the solution has been reached, STOP. Otherwise, update counters: if $j < T$ then, $j \leftarrow j+1$; else, if $j = T$, then $j = 1$, $IT \leftarrow IT + 1$ and go back to Step 1.

The consistency between the market-clearing price and the O-GENCO quota is explicitly enforced while solving problems (4)–(8) through constraints (7) and (8).

A flowchart of this algorithm is shown in Fig. 1.

Two computational considerations are in order.

1) The number of steps of the price–quota function may change from hour to hour, reflecting differences in hourly demands. This is taken into account in step 1 of the above algorithm.
2) If the number of steps of the price–quota function is large, a window strategy can be used so that only the neighboring steps of the current step are considered. This results in lower computational burden without typically altering the quality of the solution attained.

After fixing market-clearing price values the resulting problem (4)–(8) can be efficiently solved because it is a mixed-integer linear programming problem with a moderate

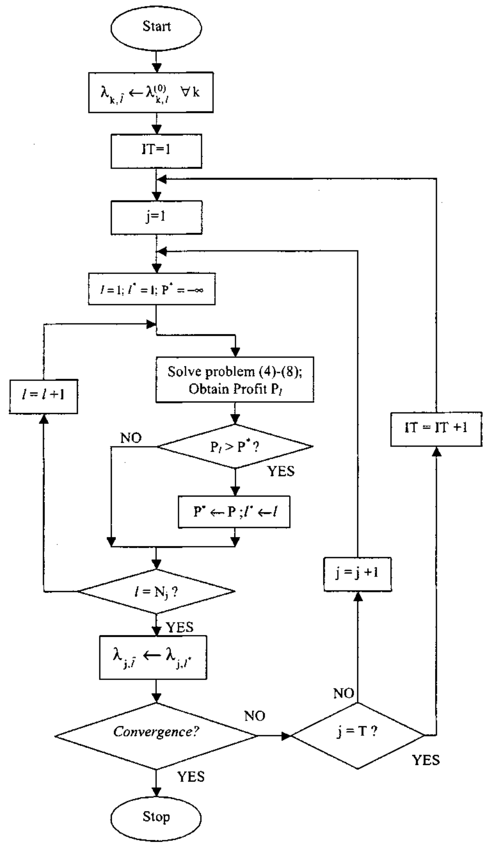

Fig. 1. Solution technique flowchart.

TABLE I
GENERATING UNITS DATA

Type	A	B	C	D	E	F	G
OG / RU	3 / 9	3 / 9	3 / 9	3 / 9	3 / 8	3 / 8	2 / 8
$\overline{P}$ [MW]	12	76	100	155	197	350	400
$\underline{P}$ [MW]	2.4	15.2	25	54.25	68.95	140	100
C_1 (*)	29.4	21.8	23.4	18.9	24.1	19.2	13.5
C_2 (*)	29.8	22.7	25.2	19.5	25.5	20.3	13.6
C_3 (*)	33.7	26.4	27.2	20.3	26.6	21.1	14.0
C_4 (*)	38.2	30.4	28.5	21.4	27.8	22.3	14.4
RR [MW/h]	12	76	100	155	180	120	400
SC [$]	196	1353	1635	2173	2239	10190	NA (**)
MUT [h]	4	8	8	8	12	24	NA
MDT [h]	2	4	8	8	10	48	NA

(*) Units: [$/MWh] (**) NA: not applicable

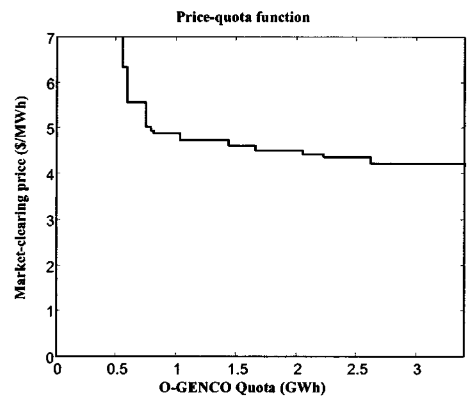

Fig. 2. Price–quota curve for the hour of highest demand.

number of binary variables. Available software to address this problem includes CPLEX [16].

It should be noted that the above algorithm is a coordinate descent algorithm (coupled with a mixed-integer linear programming algorithm), being every coordinate the multi-step (multi-value) market-clearing price at every time period.

IV. CASE STUDY

A power pool comprising 80 bidding units is considered. Data for these units are based on the 1996 IEEE RTS [17]. The considered O-GENCO owns 20 of these units ranging from base-loaded plants to peakers. Data for the O-GENCO units is provided in Table I. The remaining units act as price takers. Data for these price taker units is also provided in Table I.

In this table, "Type" indicates the unit type; "OG/RU" provides the number of units belonging to the O-GENCO and the remaining number of units, respectively; "$\overline{P}$" and "$\underline{P}$" are the capacity and the minimum power output, respectively; "C_1," "C_2," "C_3," and "C_4" are respectively the marginal cost in $/MWh for the first, second, third, and fourth power blocks of every unit; "RR" is the up and down ramp limit; "SC" is the start-up cost; and "MUT" and "MDT" are the minimum up and down time, respectively. A planning horizon of 1-day hour by hour is considered.

The O-GENCO price–quota curve for every hour is obtained by simulating the pool market-clearing mechanism and assuming that all units (including those belonging to the O-GENCO) bid at their corresponding marginal cost. A 4-block bidding curve is assumed for every unit. After bidding, the market is cleared using an economic dispatch algorithm (simple auction mechanism). It should be noted that alternative procedures can be used to obtain or estimate price–quota curves. The 12-step price–quota curve for the hour of the highest demand is shown in Fig. 2.

The problem described is solved using the proposed coordinate descent algorithm coupled with a mixed-integer linear programming solver. For the reported case study the whole planning horizon was considered three times, using windows of different sizes, and resulting in the solution of 405 problems. Using CPLEX under GAMS [16], total computing time in a PENTIUM-based PC with 132 MB of RAM was 2 h and 16 min. The maximum benefit for the O-GENCO is $423 182.

For every hour, the demand, the optimal production of the O-GENCO as a whole and the market-clearing price is shown in Fig. 3.

Fig. 4 depicts the hourly production of the O-GENCO against its competitors. The following comments are in order. During hours 11–14, 16–22 the O-GENCO does exercise its market power lowering its production and therefore raising the price, which results in higher total profits for the O-GENCO. During the remaining hours, and due to a relatively low demand, the O-GENCO has no room to exercise its market power to its own benefit and therefore it behaves mostly as a price taker.

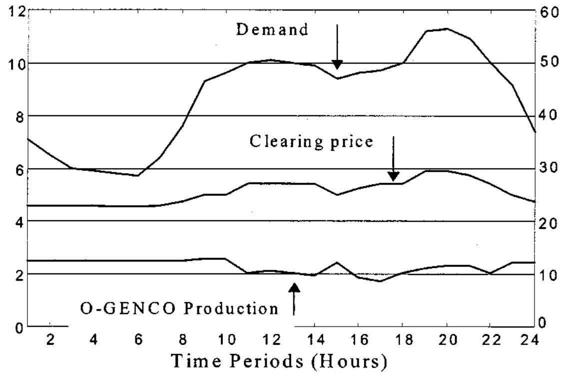

Fig. 3. Demand and O-GENCO optimal production in [GWh] (left vertical axis). Market-clearing price in [$/MWh] (right vertical axis).

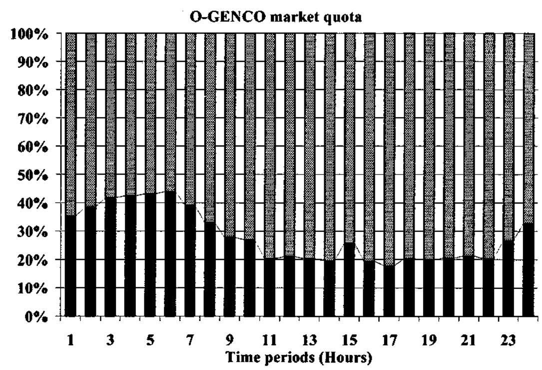

Fig. 4. O-GENCO market–quota (black).

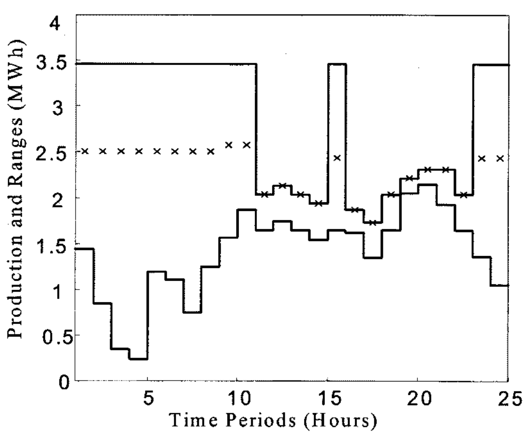

Fig. 5. O-GENCO production and production range to maintain best prices.

Fig. 5 further illustrates the oligopolistic behavior of the O-GENCO. This figure shows the actual production for every hour (x) and the range of production of the O-GENCO (stairs plots) to maintain its most favorable price. Note that in each of the previously mentioned hours the O-GENCO produces the maximum power which allows it to keep its most favorable price. If these most favorable prices are changed (increased or decreased), the O-GENCO total benefit decreases.

V. Conclusion

This paper provides a rigorous modeling of the maximum profit problem faced by an oligopolistic generating company in a competitive electric power market built around a pool. The price–quota curve of the oligopolistic generating company is stepwise and therefore discontinuous. Thus, the resulting maximum profit problem is discontinuous, nonlinear and mixed-integer. Its structure suggests a coordinate-descent solution procedure coupled with mixed-integer linear programming. This solution technique, which has been efficiently implemented, has been tested in quite a few different case studies and proved efficient and accurate.

Appendix A

The running cost of a thermal unit and its technical constraints are described in this Appendix. The running cost $c_i(k)$ is expressed as

$$c_i(k) = A_i v_i(k) + vc_i(k) + UC_i y_i(k) + DC_i z_i(k) \quad \forall i \in \Omega, \quad \forall k \in K \tag{A1}$$

where A_i represents the fixed cost of unit i [$/h], $vc_i(k)$ is the piecewise linear variable cost [$/h], UC_i denotes the start-up cost of unit i [$], DC_i is the shut-down cost of unit i [$], $v_i(k)$ is a 0/1 variable which is equal to 1 if unit i is on-line in period k, $y_i(k)$ denotes the 0/1 variable which is equal to 1 if unit i is started-up at the beginning of period k, and $z_i(k)$ is the 0/1 variable which is equal to 1 if unit i is shut-down at the beginning of period k.

Equation (A1) express the running cost of unit i in period k as the sum of a fixed term, different from zero if the unit is committed, plus the variable cost, the start-up cost, and the shut-down cost.

The piecewise linear variable cost $vc_i(k)$ is formulated as

$$vc_i(k) = \sum_{n=1}^{NL} F_n(i) b_n(i, k) \qquad \forall i \in \Omega, \quad \forall k \in K \tag{A2}$$

$$p_i(k) = \underline{P}_i v_i(k) + \sum_{n=1}^{NL} b_n(i, k) \quad \forall i \in \Omega, \quad \forall k \in K \tag{A3}$$

$$0 \le b_n(i, k) \le \overline{b}_n(i) \quad \forall n \in N, \quad \forall i \in \Omega, \quad \forall k \in K \tag{A4}$$

where $b_n(i, k)$ represents the power produced by unit i in period k using the nth power block [MW], $\overline{b}_n(i)$ is the size of the nth power block of unit i [MW], $\underline{P}_i$ is the minimum power output of unit i [MW], NL is the number of blocks of the piecewise linear variable cost function, $F_n(i)$ denotes the slope of block n of the variable cost of unit i [$/MWh], and N is the set of power blocks.

Constraints (A2) express the variable cost of unit i in period k as the sum of the corresponding terms of the piecewise linearization. Constraints (A3) state that the power output of unit i in period k is the sum of the power generated using each block plus the minimum power output. Constraints (A4) set the of the power generated in each block. This power should be greater than 0 and less than the size (in MW) of each block. This formulation assumes that the cost is monotonically increasing. Nonconvex costs can be easily modeled by using additional binary variables [18].

The set of production constraints of power unit i Π_i is presented in the following[19], [20]:

$$\underline{p}_i(k) \le p_i(k) \le \overline{p}_i(k) \qquad \forall i \in \Omega, \quad \forall k \in K \tag{A5}$$

$$\begin{aligned}\overline{p}_i(k) = \mathrm{Min}\,\{&\overline{P}_i\,[v_i(k) - z_i(k+1)] \\ &+ z_i(k+1)SD_i,\, p_i(k-1) + RU_i v_i(k-1) \\ &+ y_i(k)SU_i\} \qquad \forall i \in \Omega, \quad \forall k \in K\end{aligned} \tag{A6}$$

$$\begin{aligned}\underline{p}_i(k) = \mathrm{Max}\,\{&\underline{P}_i v_i(k),\, [p_i(k-1) - RD_i]\, v_i(k)\} \\ &\forall i \in \Omega, \quad \forall k \in K\end{aligned} \tag{A7}$$

$$\begin{aligned}&[x_i(k-1) - UT_i]\,[v_i(k-1) - v_i(k)] \ge 0 \\ &\quad \forall i \in \Omega, \quad \forall k \in K\end{aligned} \tag{A8}$$

$$\begin{aligned}&[x_i(k-1) + DT_i]\,[v_i(k) - v_i(k-1)] \le 0 \\ &\quad \forall i \in \Omega, \quad \forall k \in K\end{aligned} \tag{A9}$$

$$\begin{aligned}&y_i(k) - z_i(k) = v_i(k) - v_i(k-1) \\ &\quad \forall i \in \Omega, \quad \forall k \in K\end{aligned} \tag{A10}$$

$$y_i(k) + z_i(k) \le 1 \qquad \forall i \in \Omega, \quad \forall k \in K \tag{A11}$$

$$v_i(k),\, y_i(k),\, z_i(k) \in \{0, 1\} \qquad \forall i \in \Omega, \quad \forall k \in K \tag{A12}$$

where DT_i is the minimum down time of unit i [h], $\overline{P}_i$ the capacity of unit i [MW], RD_i the ramp-down limit of unit i [MW/h], RU_i the ramp-up limit of unit i [MW/h], SD_i the shut-down ramp limit of unit i [MW/h], SU_i the start-up ramp limit of unit i [MW/h], UT_i the minimum up time of unit i [h], $\overline{p}_i(k)$ the maximum available power output of unit i in period k [MW], $\underline{p}_i(k)$ the minimum available power output of unit i in period k [MW], $x_i(k)$ the number of periods unit i has been on (+) or off (−) at the end of period k. For unit consistency, it should be noted that time periods of 1 h are considered.

Constraints (A5) force every thermal unit to work below its maximum available power output, and above its minimum available power output. Constraints (A6) state that the maximum available power output in every period depends on ramp rate limits. Constraints (A7) update the minimum available power output taking into account the ramp rate limits. Constraints (A8) and (A9) enforce feasibility in terms of minimum up and minimum down time constraints, respectively. The remaining constraints preserve the logic of running, start-up, and shut-down status changes.

From a mathematical point of view, the above formulation of the problem is mixed-integer and nonlinear. However, a linear formulation of Π_i, which has been presented recently in [18], makes it possible to solve this problem using a mixed-integer linear programming technique. It is not reproduced here for lack of space.

APPENDIX B

An example to clarify the algorithm described in Section III is provided below. For this example, an O-GENCO including two generators and a time-horizon of three hours is considered. Technical data regarding the two units is shown in Table II. Note that the running cost of each unit is represented by using only one linear block. For the sake of clarity, start-up cost, minimum up times and minimum down times are not considered.

TABLE II
SMALL EXAMPLE: GENERATING UNITS DATA

UNITS	$\overline{P}$ [MW]	$\underline{P}$ [MW]	Cost [$/MWh]	Ramp limit [MW/h]
1	100	30	28	50
2	200	80	22	50

TABLE III
SMALL EXAMPLE: PRICE–QUOTA CURVES DATA

	HOUR 1		HOUR 2		HOUR 3	
	price	quota	price	quota	price	quota
Block 1	40	100	40	130	40	180
Block 2	38	150	36	180	35	260
Block 3	35	270	33	230	30	280
Block 4	25	300	25	300	25	300

TABLE IV
SMALL EXAMPLE: EVOLUTION OF THE ALGORITHM

		Values of λ for which the problem is solved					
IT	j	$\lambda^{(1)}$	$\lambda^{(2)}$	$\lambda^{(3)}$	$\lambda^{(4)}$	λ	P^*
1	1	40,40,40	38,40,40	35,40,40	25,40,40	**38,40,40**	**7980**
1	2	38,40,40	38,36,40	38,33,40	38,25,40	**38,36,40**	**8160**
1	3	38,36,40	38,36,35	38,36,30	38,36,25	**38,36,40**	**8160**
2	1	40,36,40	38,36,40	35,36,40	25,36,40	**35,36,40**	**8490**
2	2	35,40,40	35,36,40	35,33,40	35,25,40	**35,33,40**	**8500**
2	3	35,33,40	35,33,35	35,33,30	35,33,25	**35,33,40**	**8500**
3	1	40,33,40	38,33,40	35,33,40	25,33,40	**35,33,40**	**8500**
3	2	35,40,40	35,36,40	35,33,40	35,25,40	**35,33,40**	**8500**
The algorithm ends here.							

Table III shows the price–quota curves for the three hours. Note that prices are in [$/MWh] and quota-blocks of the O-GENCO in [MW]. Regarding hour 1, Table III is interpreted as: if production of the O-GENCO is between 0 and 100 MW, the price obtained in the market is $40/MWh; if production of the O-GENCO is between 100.1 and 150 MW, the price obtained in the market is $38/MWh; and so on.

In this example, the algorithm of Section III works as follows.

Step 0) Initialize iteration counter $IT = 1$. Initialize hour counter $j = 1$. Use the first price from the price–quota curve of each hour to initialize price vector: $\lambda =$ (40, 40, 40). Initialize best step-number $l^* = 1$. Initialize best Profit $P^* = -\infty$.

Step 1) Solve problem (4)–(8) for all possible price vectors obtained by substituting the values of the jth (1st) element in λ with all possible values defined at the corresponding price–quota curve; that is, solve problem (4)–(8) for four different price vectors: $\lambda^{(1)} =$ (40, 40, 40), $\lambda^{(2)} =$ (38, 40, 40), $\lambda^{(3)} =$ (35, 40, 40), and $\lambda^{(4)} =$ (25, 40, 40).

Step 2) Choose the best value of P as the new P^*. In this case, the highest profit was obtained for $\lambda^{(2)} =$ (38, 40, 40), and P^* is $7980. The price vector is updated as: $\lambda =$ (38, 40, 40).

Step 3) Last time that the profit improved in the previous step, therefore, the algorithm should continue; update $j = 1 + 1 = 2$, and go to Step 1.

Step 1) Solve problem (4)–(8) for all possible values of the jth (2nd) price; that is, solve problem (4)–(8) with four different price vectors: $\lambda^{(1)} =$ (38, 40, 40), $\lambda^{(2)} =$ (38, 36, 40), $\lambda^{(3)} =$ (38, 33, 40) and $\lambda^{(4)} =$ (38, 25, 40).

Step 2) Choose the best value of P as the new P^*. In this case, the highest profit was obtained using $\lambda^{(2)} =$ (38, 36, 40); P^* becomes \$8160. λ is updated as: $\lambda =$ (38, 36, 40).

The algorithm continues as stated in Table IV. The optimal solution is obtained in iteration 3. Optimal prices are \$35/MWh, \$33/MWh and \$40/MWh, respectively. Optimal profit is \$8500 and optimal productions for the O-GENCO are 270 MW, 230 MW, and 180 MW, respectively.

REFERENCES

[1] R. Green and D. Newbery, "Competition in the British electricity spot market," *J. Political Econ.*, vol. 100, no. 5, pp. 929–953, Oct. 1992.

[2] J. J. González and P. Basagoiti, "Spanish power exchange market and information system design concepts, and operating experience," in *Proc. IEEE 21st Int. Conf. Power Industry Comput. Applicat.*, Santa Clara, CA, May 1999, pp. 245–252.

[3] G. B. Sheblé, *Computational Auction Mechanisms for Restructured Power Industry Operation*. Norwell, MA: Kluwer, 1999.

[4] D. G. Luenberger, *Linear and Nonlinear Programming*, Second ed. Reading, MA: Addison-Wesley, 1989.

[5] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*. New York: Wiley, 1988.

[6] R. Green, "The electricity contract market," in *Proc. Second Power Conf.* Berkeley, CA: Univ. California Energy Inst., Mar. 1997.

[7] J. Contreras, O. Candiles, J. I. de la Fuente, and T. Gómez, "Auction design in day-ahead electricity markets," *IEEE Trans. Power Syst.*, vol. 16, pp. 88–96, Feb. 2001.

[8] S. Hao and G. Angelidis, "Consumer payment minimization in power pool auction," in *Proc. IEEE 20th Int. Conf. Power Industry Comput. Applicat.*, Columbus, OH, May 1997, pp. 368–373.

[9] H. Rudnick, R. Varela, and W. Hogan, "Evaluation of alternatives for power system coordination and pooling in a competitive environment," *IEEE Trans. Power Syst.*, vol. 12, pp. 605–613, May 1997.

[10] T. Alvey, D. Goodwin, X. Ma, D. Streiffert, and D. Sun, "A security-constrained bid-clearing system for the New Zealand wholesale electricity market," *IEEE Trans. Power Syst.*, vol. 13, pp. 340–346, May 1998.

[11] S. Makkonen and R. Lahdelma, "Analysis of power pools in the deregulated energy market through simulation," in *Proc. 32nd Annu. Hawaii Int. Conf. Syst. Sci.*, Hawaii, Jan. 1999.

[12] S. Borenstein and J. Bushnell, "An empirical analysis of the potential for market power in California's electricity industry," Univ. California Energy Inst., Berkeley, CA, Power Working Paper PWP-044r, Mar. 1997.

[13] C. Wolfram, "Strategic bidding in a multi-unit auction: An empirical analysis of bids to supply electricity in England and Wales," in *Proc. Second Power Conf.* Berkeley, CA: Univ. California Energy Inst., Mar. 1997.

[14] J. García, J Román, J. Barquín, and A. González, "Strategic bidding in deregulated power systems," in *Proc. 13th Power Syst. Computat. Conf.*, Trondheim, Norway, July 1999, pp. 258–264.

[15] S. Borenstein, J. Bushnell, and C. R. Knittel, "Market power in electricity markets: Beyond concentration measures," Univ. California Energy Inst., Berkeley, CA, Power Working Paper PWP-059r, Jan. 1999.

[16] A. Brooke, D. Kendrick, and A. Meeraus, "GAMS/CPLEX 6.5.2 user notes," GAMS Development Corp., Washington, D.C., 1996.

[17] Reliability Test System Task Force, "The IEEE reliability test system 1996," *IEEE Trans. Power Syst.*, vol. 14, pp. 1010–1020, Aug. 1999.

[18] J. M. Arroyo and A. J. Conejo, "Optimal response of a thermal unit to an electricity spot market," *IEEE Trans. Power Syst.*, vol. 15, pp. 1098–1104, Aug. 2000.

[19] S. M. Shahidehpour and S. K. Tong, "An overview of power generation scheduling in the optimal operation of a large scale power system," *Elect. Mach. Power Syst.*, vol. 19, pp. 731–762, 1991.

[20] T. S. Dillon, K. W. Edwin, H. D. Kochs, and R. J. Tand, "Integer programming approach to the problem of optimal unit commitment with probabilistic reserve determination," *IEEE Trans. Power Apparat. Syst.*, vol. PAS-97, pp. 2154–2166, Nov./Dec. 1978.

Antonio J. Conejo (S'86–M'91–SM'98) received the B.S. degree from the Universidad P. Comillas, Madrid, Spain, the S.M. degree from the Massachusetts Institute of Technology (MIT), Cambridge, MA, and the Ph.D. degree from the Royal Institute of Technology, Stockholm, Sweden, in 1983, 1987, and 1990, respectively, all in electrical engineering.

He is currently Professor of Electrical Engineering at the Universidad de Castilla-La Mancha, Ciudad Real, Spain. His research interests include restructuring, control, operations, planning and economics of electric energy systems, as well as optimization theory and its applications.

Javier Contreras (M'98) was born in Zaragoza, Spain, in 1965. He received the B.S. degree in electrical engineering from the University of Zaragoza, Zaragoza, Spain, the M.Sc. degree in electrical engineering from the University of Southern California, Los Angeles, and the Ph.D. degree in electrical engineering from the University of California, Berkeley, in 1989, 1992, and 1997, respectively.

His research interests include power systems planning, operations and economics, and electricity markets. He is currently an Assistant Professor at the Universidad de Castilla-La Mancha, Ciudad Real, Spain.

José Manuel Arroyo (S'96) received the Ingeniero Industrial degree from the Universidad de Málaga, Málaga, Spain, in 1995. He is currently pursuing the Ph.D. degree in power system operations planning from the Universidad de Castilla-La Mancha, Ciudad Real, Spain.

He is currently Assistant Professor of Electrical Engineering at the Universidad de Castilla-La Mancha. His research interests include operations, planning and economics of electric energy systems, as well as optimization and parallel computation.

Sebastián de la Torre (S'00) received the Ingeniero Industrial degree from the Universidad de Málaga, Málaga, Spain, in 1999. He is currently pursuing the Ph.D. degree in power systems operations planning at the Universidad de Castilla–La Mancha, Ciudad Real, Spain.

His research interests include operations and economics of electrical energy systems, restructuring of electric systems, and development of new electric markets.