A Statistical Model for Indoor SISO PLC Channels

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***Abstract*—This paper proposes a statistical channel model for indoor single-input single-output (SISO) power line communications (PLC) in the 2-80 MHz frequency band. The model follows a purely top-down strategy, since it makes no physical assumptions about the underlying power network. The frequency response is modeled as a multivariate random variable (RV) whose parameters are derived from the statistics of a set of 458 channels measured in different European countries. To this end, we firstly assess the log-normality of the amplitude response and discuss the difficulties associated to the estimation of the covariance matrix of the amplitude response at different frequencies, highlighting the need for using a regularization method. The parameters of the model are then derived by approximating the statistics of the measured channels by means of novel analytical expressions. Finally, the validity of the proposal is evaluated by comparing the average channel gain, the delay spread and the coherence bandwidth of the channels generated according to it with the ones of the measured channels and of channels generated using other top-down statistical models proposed in the literature.**

***Index Terms*—power line communications, channel model, top-down, statistical, log-normal**

I. Introduction

Channel modelling is of utmost importance for the design and performance assessment of communication systems. Two main modelling approaches are employed for this purpose. The one referred to as top-down defines a signal model based on the features of the signal propagation in the considered scenario and derives the parameters of the model from measurements. Alternatively, the bottom-up strategy models the underlying physical structure of the propagation environment. This latter approach has been successfully employed in power line communications (PLC) by modelling the indoor power grid as a set of interconnected transmission lines [1] [2]. Bottom-up models can estimate the channel frequency response (CFR) without the need for measurements and can be also used to obtain statistically representative channels by generating topologies in a random manner [3] [4]. However, their main drawback is the need for precise knowledge of the network topology and the cable parameters, which notably differ among countries.

Top-down models have been less commonly employed in PLC. Given the multipath nature of PLC channels, the first approaches of this category modelled the CFR as a set of delayed and weighted echoes. The work in [5] represented outdoor PLC channels in the frequency band up to 20 MHz. However, it is not a statistical model because its parameters are deterministically selected to represent the CFR corresponding to links of different lengths. Based on this framework, the model in [6] proposed statistical distributions for the model parameters. However, the assumed distributions are not empirically supported. A similar approach is followed in [7], but based on measurements trials. A particularly simple statistical top-down model of this class was proposed in [8]. Nevertheless, it was unable to model well-known features of indoor PLC channels such as the relation between the average channel gain and the delay spread [9].

The work in [10] suggested that the average channel gain and the delay spread of PLC channels can be modelled as log-normal random variables (RVs). However, the actual distribution of the amplitude of the CFR at each frequency is still controversial. Hence, while [7] shows that the log-normal distribution gives good fitting to some measured channels, the hypothesis tests reject this end, to a great extend, for the measured channels in [11], although the log-normal distribution gives larger likelihood values than other distributions commonly employed in channel modelling. Based on this fact, Pittolo and Tonello [12] proposed a multiple-input multiple-output (MIMO) model in which the amplitude of the CFR is modeled as a multivariate log-normal RV and the unwrapped phase of the frequency response is assumed to be linear with a slope generated according to a generalized extreme value distribution.

Following a similar approach to the one in [12], this article proposes a top-down statistical channel model for indoor single-input single-output (SISO) PLC in the frequency band up to 80 MHz. The following contributions are made:

- We show that the correlation between the amplitude of the CFR at two frequencies is notably larger in the high frequency range than in the low one. Accordingly, a novel approximation of the correlation matrix that takes this fact into account is proposed.
- We exploit the correlation between the average channel gain and the slope of the unwrapped phase of the CFR to generate realistic phases of the modeled responses.
- We discuss some practical aspects such as the difficulty associated to the estimation of large covariance matrices and prevents from using the Kolmogorov-Smirnov test to determine the most appropriate analytical approximation of some model parameters.

The rest of this paper is organized as follows. Section II analyzes the features of the set of measured channels

TABLE I
RESULTS OF DIFFERENT NORMALITY TESTS APPLIED TO THE AMPLITUDE OF THE CFR IN DECIBELS. PERCENTAGE OF CASES WHERE THE NORMALITY IS REJECTED ($\%R$) AND CORRESPONDING AVERAGE P-VALUES

	Set 1		**Set 2**		**Set 3**		**Overall set**	
	% R	**mean p-value**	**% R**	**mean p-value**	**% R**	**mean p-value**	**% R**	**mean p-value**
Lilliefors	19.03	0.25	6.42	0.38	57.10	0.08	50.98	0.02
Shapiro-wilk	46.55	0.17	10.23	0.28	82.95	0.03	88.02	0.05
Jarque-Bera	14.43	0.23	0.48	0.30	50.28	0.10	65.23	0.06
Chi-Square	18.79	0.29	6.58	0.38	58.29	0.10	56.93	0.10
Anderson-Darling	33.07	0.20	8.49	0.33	76.37	0.04	70.63	0.12
Kolmogorov-Smirnov	0.56	0.60	0.00	0.72	1.74	0.36	5.40	0.40

employed to derive the model parameters. Section III presents the proposed model and the analytical expressions used to approximate its parameters from the set of measured channels. The model is validated in Section IV. Finally, Section V summarizes the main aspects of the work.

II. STATISTICAL ANALYSIS OF INDOOR PLC CHANNELS

The parameters of the proposed model are derived from a collection of $L = 458$ indoor CFRs measured in different European countries using a vector network analyzer. They can be grouped into three sets according to the country and the employed coupling circuit. Channels in Set 1 (166) and Set 2 (46) were all measured in Spain, although using different coupling circuits. Channels in Set 3 (246) were measured in Germany (74), Belgium (51), France (80) and the United Kingdom (41). They all use the same coupling circuit, but different from the one in Set 1 and Set 2. Since the type of cables and the network topology may vary from country to country, the heterogeneity of the measurements makes the whole set more statistically representative. The selected frequency band ranges from 1.8 to 80 MHz with a resolution of $\Delta f = 61.875$ kHz, resulting in $N = 1264$ frequency samples.

A. Probability distribution of the amplitude response

Assuming that the amplitude of the CFR is log-normally distributed, the resulting values in dB scale must be normally distributed. To assess this, several hypothesis tests have been performed. They are used to validate the null hypothesis that the logarithmic version of the amplitude CFR at each frequency is normally distributed at a significance level $\alpha = 0.05$. The Kolmogorov-Smirnov, Lillie-Fors, Shapiro-Wilk, Jarque-Bera and Anderson-Darling tests have been employed to this end. Table I shows the obtained results when applied to each set of measurements and also to the overall set. Hence, the number of samples tested in each set is N times the corresponding number of channels of the set.

As seen, the p-value is much higher for the Kolmogorov-Smirnov test than in the others in all measurement sets. Accordingly, this test yields the lowest percentage of rejection in all sets. For this test, not only the sample of sets to be tested but also the parameters of the distribution must be given. However, it is well-known that, when these parameters are derived from the same set to be tested, the p-value is overestimated [13]. Hence, it is inappropriate for this purpose.

All other tests support log-normality in Set 1 and Set 2, but neither in Set 3 nor in the overall set. This may be due to the fact that channels in Set 3 have been measured in different countries. Therefore, the parameters of the log-normal distribution may be different from country to country. The problem is that the number of channels measured in each country is not large enough to test this hypothesis. To overcome this end, the cumulative distribution function (CDF) of the kurtosis and skewness of the CFR amplitude in dB scale is computed and compared to the ones of normally distributed vectors with the same number of samples. It has been found that only about 14% of the kurtosis values computed from measurements are between the 10-th and 90-th percentiles of the kurtosis of the normal vectors. This percentage goes up to 48.5% in the case of skewness. Nevertheless, it can be conjectured that the particularly low percentage obtained for kurtosis might be largely caused by factors related to the measurement procedure (e.g., values at right the tail of the distribution might not be accurately measured because of the sensitivity of the vector analyzer or the low signal to noise ratio of the measurement).

The performed tests support the log-normality of the samples in Set 2 and, likely in Set 1, but seem to reject this hypothesis in Set 3 or, at best, is inconclusive. However, it has been verified that the log-normal distribution gives a larger value of the likelihood function than other distributions commonly used in channel modelling such as the exponential, gamma, normal, Rayleigh, Weibull and log-logistic, which is in agreement with other published results [14]. Accordingly, the log-normal distribution is assumed in the proposed model.

B. Probability distribution of the phase response

In order to analyse the phase distribution, we first perform the Kolmogorov-Smirnov test on the phase values of the measurements at each frequency under the null hypothesis that they are uniformly distributed in the interval $[-\pi, \pi)$. Since the parameters of the distribution are not derived from the sample to be tested, the overestimation of the p-value highlighted in the previous subsection does not occur now. The percentage of frequencies in which the null hypothesis is rejected at α= 0.05 is 16.56%. The average p-value is 0.31.

Additionally, unbiased estimates of the mean and variance at each frequency value are computed and the corresponding CDFs have been compared to the ones derived from a large set of uniformly distributed vectors with the same number of samples as the measured channels. About 71% of the mean values obtained from measurements are between 10-th and 90-th percentiles of the mean computed from the uniformly distributed vectors. This percentage is 65% for the variance. Hence, the uniform distribution of the phase in the interval $[-\pi, \pi)$ can be reasonably assumed.

C. Covariance matrix of the amplitude response

We use the conventional algebraic notation in which matrices are denoted using bold capital letters and vectors using bold lower-case. Hence, the vector with the amplitude of the CFR at frequencies $f_k = f_1+(k-1)\Delta f$, with $f_1 = 1.8$ (MHz) and $1 \leq k \leq N$, will be denoted as $\mathbf{g} = [g_1, \ldots, g_N]^T$ and its natural logarithmic version as $\mathbf{g}^{\ln} = [\ln g_1, \ldots, \ln g_N]^T$, where $(\cdot)^T$ denotes the transpose operator.

While the covariance matrix of $\mathbf{g}$, $\mathbf{C}_{\mathbf{g}}$, can be directly estimated from the measurements, we have verified that lower estimation error is obtained if the covariance matrix of $\mathbf{g}^{\ln}$, $\mathbf{C}_{\mathbf{g}^{\ln}}$, is estimated and then $\mathbf{C}_{\mathbf{g}}$ is computed as [15]

$$\mathbf{C}_{\mathbf{g}} = \operatorname{diag}(\boldsymbol{\mu}_{\mathbf{g}}) \left(e^{\mathbf{C}_{\mathbf{g}^{\ln}}} - \mathbf{1}_{N\times N} \right) \operatorname{diag}(\boldsymbol{\mu}_{\mathbf{g}}), \tag{1}$$

where $\operatorname{diag}(\mathbf{x})$ denotes a diagonal matrix obtained from vector $\mathbf{x}$ and

$$\left[\boldsymbol{\mu}_{\mathbf{g}}\right]_k = e^{\left[\boldsymbol{\mu}_{\mathbf{g}^{\ln}}\right]_k + \frac{1}{2}\left[\mathbf{C}_{\mathbf{g}^{\ln}}\right]_{kk}}, \tag{2}$$

where $[\mathbf{x}]_i$ denotes the i-th element of the vector $\mathbf{x}$, $[\mathbf{X}]_{ij}$ denotes the element on the i-th row and j-th column of the matrix $\mathbf{X}$ and $\boldsymbol{\mu}_{\mathbf{g}^{\ln}} = \mathbb{E}\left[\mathbf{g}^{\ln}\right]$.

Under the assumption that $\mathbf{g}$ is a log-normal RV, $\mathbf{g}^{\ln}$ should be normally distributed. In these circumstances, the sample covariance matrix given by

$$\widehat{\mathbf{C}}_{\mathbf{g}^{\ln}} = \frac{1}{L}\sum_{\ell=1}^{L} \mathbf{g}_\ell^{\ln} \left(\mathbf{g}_\ell^{\ln}\right)^H - \left(\frac{1}{L}\sum_{\ell=1}^{L} \mathbf{g}_\ell^{\ln}\right)\left(\frac{1}{L}\sum_{\ell=1}^{L} \mathbf{g}_\ell^{\ln}\right)^H, \tag{3}$$

where $\mathbf{g}_\ell^{\ln}$ denotes the natural logarithm of the amplitude of the ℓ-th measured CFR, is the optimum maximum likelihood (ML) estimator of $\mathbf{C}_{\mathbf{g}^{\ln}}$.

However, the estimation of covariance matrices using (3) is problematic when both the sample size and the matrix dimension are high, since large eigenvalues tend to be overestimated and small ones to be underestimated. Furthermore, negative eigenvalues appear, resulting in a non-positive definite matrix. To overcome this issue, the matrix regularization method proposed in [16] is employed in this work. This ensures the positive-definiteness of the estimated matrix by estimating a banded version of its inverse. The size of the band is set to 425, which has been empirically proven to be the highest value that satisfies the positive-definiteness constraint.

It is interesting to analyze the correlation of the CFR amplitude at different frequencies. To this end, the correlation matrix of $\mathbf{g}^{\ln}$, computed as

$$\left[\mathbf{R}_{\mathbf{g}^{\ln}}\right]_{ij} = \frac{\left[\mathbf{C}_{\mathbf{g}^{\ln}}\right]_{ij}}{\sqrt{\left[\mathbf{C}_{\mathbf{g}^{\ln}}\right]_{ii}\left[\mathbf{C}_{\mathbf{g}^{\ln}}\right]_{jj}}}, \tag{4}$$

is depicted in dB scale in Figure 1. As seen, relatively high correlation values are observed in almost every point. Moreover, it is worth noting that the band around the main diagonal is wider at higher frequencies.

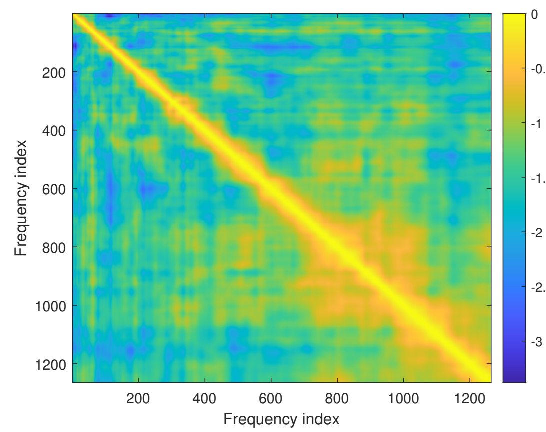

Fig. 1. Logarithmic version of the correlation matrix of the amplitude in dB scale $\mathbf{R}_{\mathbf{g}^{\ln}}$.

III. Proposed Channel Model

The CFR vector $\mathbf{h}$ can be expressed in terms of the amplitude and phase vectors, $\mathbf{g}$ and $\boldsymbol{\phi}$ as

$$\mathbf{h} = \mathbf{g} \circ e^{j\boldsymbol{\phi}}, \tag{5}$$

where $\circ$ denotes the Hadamard product. In order to simplify the modelling procedure, $\mathbf{g}$ and $\boldsymbol{\phi}$ are assumed to be independent, as in [12].

Since $\mathbf{g}$ is assumed to be log-normally distributed with mean $\boldsymbol{\mu}_{\boldsymbol{g}}$ and covariance matrix $\mathbf{C}_{\mathbf{g}}$, it can be generated from the normal RV $\mathbf{g}^{\ln} \sim \mathcal{N}\left(\boldsymbol{\mu}_{\mathbf{g}^{\ln}}, \mathbf{C}_{\mathbf{g}^{\ln}}\right)$ as $\mathbf{g} = e^{\mathbf{g}^{\ln}}$, where $\boldsymbol{\mu}_{\boldsymbol{g}}$ and $\mathbf{C}_{\mathbf{g}}$ are related to $\boldsymbol{\mu}_{\mathbf{g}^{\ln}}$ and $\mathbf{C}_{\mathbf{g}^{\ln}}$ by means of (1) and (2), respectively. Moreover, $\mathbf{g}^{\ln}$ can be related to a standard normal RV, $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$, being $\mathbf{I}_N$ the $N \times N$ identity matrix, as

$$\mathbf{g}^{\ln} = \mathbf{L}\mathbf{z} + \boldsymbol{\mu}_{\mathbf{g}^{\ln}}, \tag{6}$$

where $\mathbf{L}$ is a lower triangular matrix related to $\mathbf{C}_{\mathbf{g}^{\ln}}$ by means of the Cholesky decomposition as $\mathbf{C}_{\mathbf{g}^{\ln}} = \mathbf{L}\mathbf{L}^T$.

Besides the uniform distribution of $\boldsymbol{\phi}$ in the interval [-π, π), it is well-known that the slope of the linear approximation of the unwrapped phase values[1], referred to as phase slope and hereafter denoted as $\widehat{m}_\phi$, is positively correlated with the

[1]The unwrapped phase is obtained from the phase values in the range $[0, 2\pi)$ by adding multiples of $\pm\pi$ whenever the jump between consecutive values is greater than or equal to π.

average gain [14]. This behavior also occurs in the measured channels, as can be observed in Fig. 2, where the scatter plot of the estimated phase slope, $\widehat{m}_{\phi}$, vs the estimated average channel gain, $G = \frac{20\log_{10}(e)}{N}\sum_{k=1}^{N} g_k^{\ln}$, of the measured channels is depicted along with its optimum linear least squares (LS) fitting, $\widetilde{m}_{\phi}$, whose expression is given in (7). This behavior can be exploited to generate realizations of $\boldsymbol{\phi}$. To this end, a realization of $\mathbf{g}^{\ln}$ is firstly generated using (6). Then, the value of G is computed and the phase slope, $\widehat{m}_{\phi}$, is obtained from (7). Finally, the unwrapped values of $\boldsymbol{\phi}$ are generated as a linear function with the obtained slope and zero y-intercept.

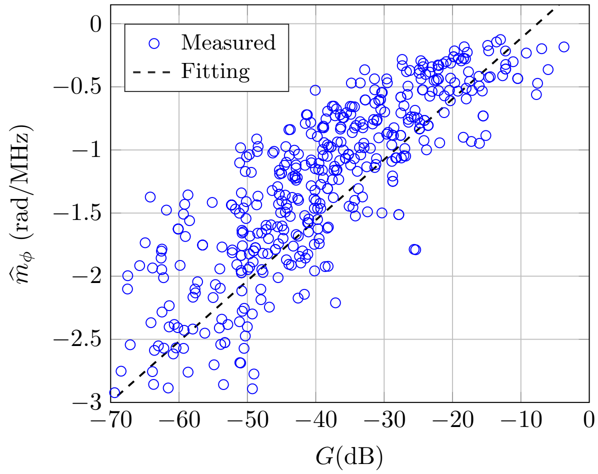

Fig. 2. Scatter plot of the slope of the unwrapped phase of the CFR and the average gain of the measured channels.

$$\widetilde{m}_{\phi} = 0.364 + 0.048 \cdot G \quad \text{(rad/MHz)} \tag{7}$$

A. Derivation of the channel model parameters

In this subsection, analytical expressions of the model parameters, $\boldsymbol{\mu}_{\mathbf{g}^{\ln}}$ and $\mathbf{C}_{\mathbf{g}^{\ln}}$, are obtained by approximating the magnitudes estimated from the measured channels.

Figure 3 shows the ML estimate of $\boldsymbol{\mu}_{\mathbf{g}^{\ln}}$, $\widehat{\boldsymbol{\mu}}_{\mathbf{g}^{\ln}} = \left[\widehat{\mu}_1^{\ln}, \ldots, \widehat{\mu}_N^{\ln}\right]^T$, along with its optimum exponential fitting given by

$$\widetilde{\mu}_k^{\ln} = a + b e^{c f_k^d (\text{MHz})}, \tag{8}$$

where $a = 11.966$, $b = -6.489$, $c = 8.166 \cdot 10^{-1}$ and $d = 3.661 \cdot 10^{-2}$.

The covariance matrix $\mathbf{C}_{\mathbf{g}^{\ln}}$ can be expressed in terms of the correlation matrix, $\mathbf{R}_{\mathbf{g}^{\ln}}$, as $\mathbf{C}_{\mathbf{g}^{\ln}} = \mathbf{R}_{\mathbf{g}^{\ln}} \boldsymbol{\sigma}_{\mathbf{g}^{\ln}} \boldsymbol{\sigma}_{\mathbf{g}^{\ln}}^T$, where $\boldsymbol{\sigma}_{\mathbf{g}^{\ln}} = \left[\sigma_1^{\ln}, \ldots, \sigma_N^{\ln}\right]^T$ is the standard deviation vector given by $\sigma_k^{\ln} = \sqrt{\left[\mathbf{C}_{\mathbf{g}^{\ln}}\right]_{kk}}$.

In contrast to the values of $\widehat{\boldsymbol{\mu}}_{\mathbf{g}^{\ln}}$, the ML estimate of the standard deviation vector exhibits a less clear shape, as shown in Fig.4. Hence, the values of $\sigma_k^{\ln}$ have been approximated by a constant set to $\widetilde{\sigma}^{\ln} = 1.99$. The procedure employed to obtain this value will be described thereafter.

Regarding $\mathbf{R}_{\mathbf{g}^{\ln}}$, it has been observed that the elements of the i-th row, $1 \leq i \leq N$, can be approximated as

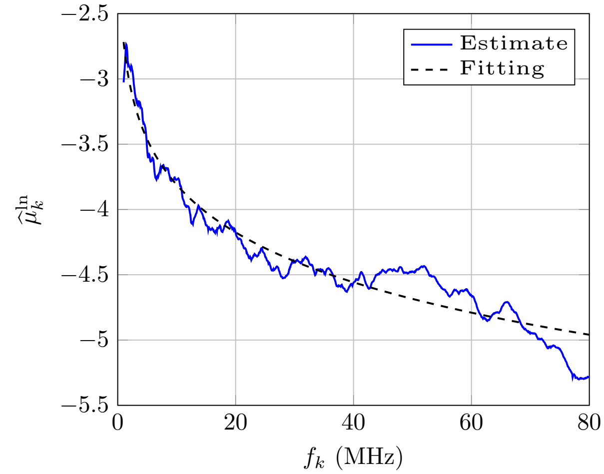

Fig. 3. ML estimate of the mean vector $\boldsymbol{\mu}_{\mathbf{g}^{\ln}}$ and its exponential fitting.

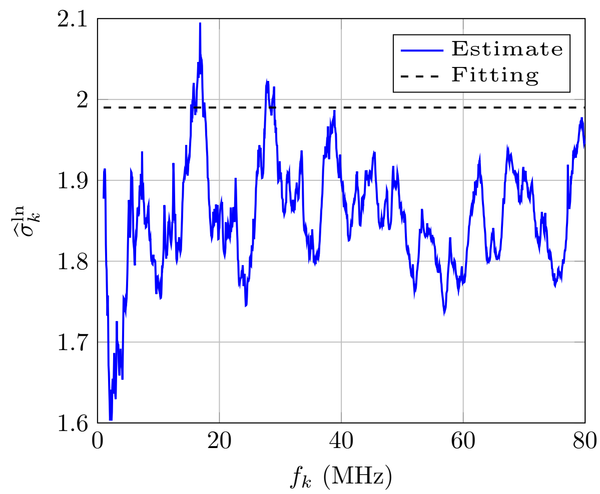

Fig. 4. Estimate of the standard deviation vector $\boldsymbol{\sigma}_{\mathbf{g}^{\ln}}$ and its constant fitting.

$$\left[\widetilde{\mathbf{R}}_{\mathbf{g}^{\ln}}\right]_{ij} = \begin{cases} 1 - \frac{1}{W_i}(j-i) & i \leq j \leq i + t_i \\ \rho_i & i + t_i < j \leq N \end{cases}, \tag{9}$$

where ρ_i is a *correlation floor* value, with $\rho_i = 0.65$ for i corresponding to frequency values up to 30 MHz and $\rho_i = 0.75$ above this frequency, and $1/W_i$ and $t_i = min\{\lfloor(1-\rho_i)\, W_i\rfloor, N - i\}$, with $\lfloor\cdot\rfloor$ denoting the floor function, are the slope and the width of the linear approximation, respectively. Since the correlation matrix is symmetric, $\left[\widetilde{\mathbf{R}}_{\mathbf{g}^{\ln}}\right]_{ji} = \left[\widetilde{\mathbf{R}}_{\mathbf{g}^{\ln}}\right]_{ij}$.

The optimum values of W_i are obtained by solving a LS optimization on each row, yielding the results given in Fig. 5. As seen, it generally increases with frequency, which is coherent with the behavior observed in Fig. 1.

The values of W_i displayed in Fig. 5 are approximated by a linear expression $\widetilde{W}_i = m_W \cdot i + n_W$. Two strategies have been followed to determine the values of m_W and n_W. The

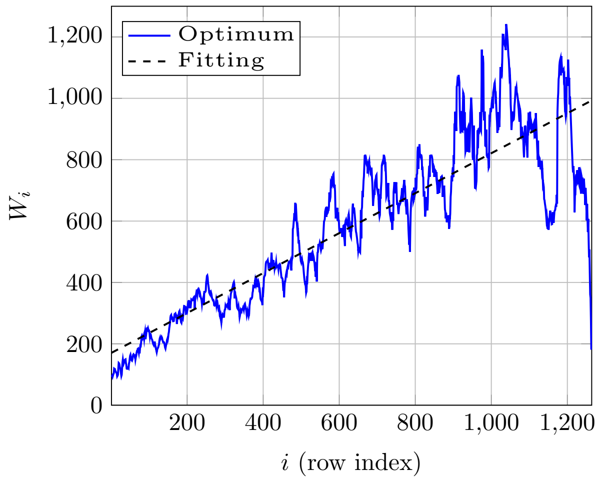

Fig. 5. Optimum width of W_i and its LS linear fitting.

first one determines these parameters from the LS fitting to the values of W_i shown in Fig. 5, yielding $m_W = 0.65$ and $n_W = 170.05$. In the second approach, $n_W = 170.05$ is selected and m_W is jointly determined with $\tilde{\sigma}^{\text{ln}}$ by minimizing the mean coherence bandwidth values of the overall set of measured channels and the generated ones, yielding $m_W = 2.01$ and $\tilde{\sigma}^{\text{ln}} = 1.99$. Since notably different values of m_W are obtained with each strategy, their optimality will be discussed in Section IV.

IV. Channel model evaluation

In this section, the proposed model is evaluated by comparing the empirical CDF of the average channel gain, the delay spread and the coherence bandwidth of the measured and generated channels. The correlation between the last two parameters is analyzed too. The magnitudes obtained with the model by Pittolo and Tonello [12] are also included in the comparison.

In order to highlight the impact of the different analytical approximations of the model parameters on the performance, three versions of the proposed model are considered. The one referred to as Model A employs the actual values of $\boldsymbol{\mu}_{\mathbf{g}^{\text{ln}}}$ and $\mathbf{C}_{\mathbf{g}^{\text{ln}}}$. While it is impractical because of the large amount of information required, it is used to obtain an upper bound of the achievable performance. The one denoted as Model B approximates the elements of $\boldsymbol{\mu}_{\mathbf{g}^{\text{ln}}}$ and $\mathbf{R}_{\mathbf{g}^{\text{ln}}}$ using the analytical expressions in (8) and (9), the latter with $m_W = 0.65$, but the actual ML estimate of $\sigma_{\mathbf{g}^{\text{ln}}}$ is employed. Finally, Model C is like Model B but $m_W = 2.01$ and all elements of the standard deviation vector are fixed to $\tilde{\sigma}^{\text{ln}} = 1.99$.

The rectangular transitions at the passband edges of the CFR artificially enlarge the channel impulse response length. Hence, a 150-th order bandpass finite impulse response (FIR) filter with lower and higher cutoff frequencies $f_{c1} = 2.0$ MHz and $f_{c2} = 79.5$ MHz, respectively, is applied to each CFR to avoid this issue.

Figure. 6 depicts the empirical CDF of the average channel gain of the measured and generated channels. While a relatively high correspondence is observed between the measured results and the ones obtained with all the models, the number of channels with high values of G is lower in the model by Pittolo and Tonello [12]. Nevertheless, this could be due to the differences between the set of measured channels used to obtain the model parameters in [12] and in this work.

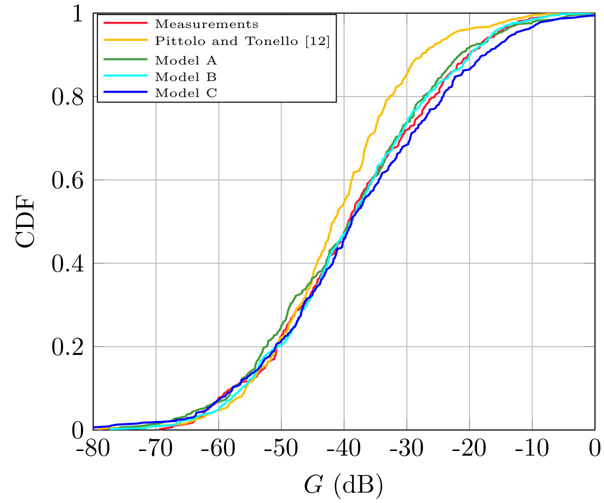

Fig. 6. Empirical CDF of the average channel gain of the measured and modelled channels. Three versions of the proposed model, named A, B and C, are compared, along with the results obtained with the model by Pittolo and Tonello [12].

Fig. 7 shows the the empirical CDF of the delay spread of the measured and generated channels. It must be mentioned that the range of delay spread values of the measured channels is similar to the ones reported in the literature [10] [14] [7]. As seen, modelled channels have a much narrower range of delay spread values. This effect is more prominent in the model by Pittolo and Tonello [12]. Interestingly, model C gives better results than model B, despite the latter uses the actual values of the standard deviation vector, which highlights the importance of the analytical approximation of the correlation matrix.

Finally, the relation between delay spread and coherence bandwidth of the generated and measured channels is depicted in Fig. 8. All cases display the well-known inverse relation between these parameters. Model A gives the closest results to the measured channels. However, it is unable to generate channels with large coherence bandwidth values (low delay spread). It can be conjectured that this might be due to the errors in the estimation of the covariance matrix. Model C is able to generate channels with larger coherence bandwidth than model A, likely because the analytical approximation of the correlation matrix compensates low correlation values caused by the errors in the estimation of the covariance matrix. However, the relation between the coherence bandwidth and the delay spread is more dissimilar to the measured channels than in model A. Finally, the model by Pittolo and Tonello [12] generates channels with the more limited range of delay spread and coherence bandwidth values.

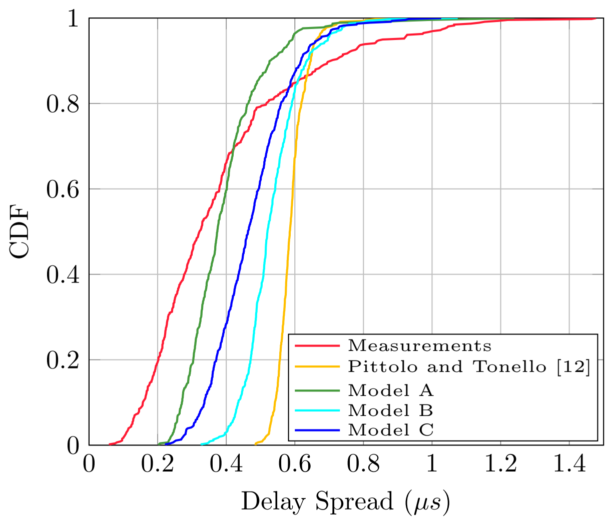

Fig. 7. Empirical CDF of the delay spread of the measured and modelled channels. Three versions of the proposed model, named A, B and C, are compared, along with the results obtained with the model by Pittolo and Tonello [12].

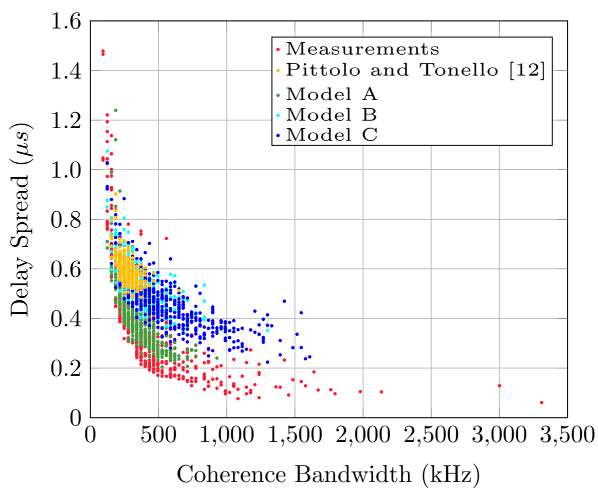

Fig. 8. Scatter plot of the delay spread vs the coherence bandwidth of the measured and modelled channels. Three versions of the proposed model, named A, B and C, are compared, along with the results obtained with the model by Pittolo and Tonello [12].

V. Conclusion

This work presents a top-down statistical channel model for indoor SISO PLC in the 2-80 MHz frequency band. The model parameters are derived from the statistics of a set of 458 channels measured in different European countries. The amplitude of the CFR is modelled as a multivariate log-normal RV, while the unwrapped phase is approximated by a linear function whose slope is obtained by exploiting the correlation between this magnitude and the average channel gain.

The parameters of the model are calculated from the statistics of the measured channels by approximating them by means of simple analytical expressions. The performance of the proposal has been assessed by comparing the statistics of the average channel gain, the delay spread and the coherence bandwidth of the measured and modelled channels.

This article also discusses some aspects of the top-down modelling approach, such as the estimation of the covariance matrix, and the use of the Kolmogorov-Smirnov test to assess the validity of the log-normal distribution of the amplitude and to determine the most appropriate analytical approximation of some model parameters.

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