

# Macroeconomics

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# Session II: Solving a DSGE model with Dynare

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- 6 Deterministic simulation
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## II.1. What is Dynare?

- Dynare is a pre-processor for Matlab/Octave. Part of Dynare is programmed in C++ and part in Matlab language. Dynare is free and covered by a GNU General Public Licence, so it can be used for both non-profit and for-profit purposes. Available for Windows, Mac and Linux platforms.
- Dynare has been developed at the CEPREMAP, by a team directed by Michel Julliard, Stéphane Adjemian and Sébastien Villemot.
- Dynare is a collection of routines and some connecting programs written by economists and computer programmers to solve, simulate, and estimate DSGE models (and more things).
- Basically, Dynare writes Matlab programs for us. One these Matlab programs has been created, we can run those without Dynare.

## II.1. What is Dynare?

- Specifically, Dynare is a pre-processor for MatLab designed to work with DSGE (backward and forward looking) and OLG models. But it can be used to work with other types of models.
- Dynare converts files \*.mod into files \*.m. Starting from a `mymodel.mod`, Dynare creates three files: `mymodel.m`; `mymodel_static.m`; `mymodel_dynamic.m`
- Dynare solves, simulate and estimate DSGE models.
- Estimation of DSGE model with maximum likelihood or Bayesian methods.

# II.1. What is Dynare?

- Alternatives to Dynare:

- gEcon: Developed in R by the Department for Strategic Analyses at the Chancellery of the Primer Minister of the Republic of Poland. Main characteristic: gEcon can solve directly a DSGE model just by writing the optimization problems for the economic agents.  
<http://gecon.r-forge.r-project.org>
- IRIS: Is a toolbox in Matlab created at the IMF. IRIS can solve, simulate and estimate (maximum likelihood methods) a DSGE model.  
<http://iris-toolbox.com>
- YADA: Is a GUI (Graphical User Interface) based Matlab program developed at the ECB. YADA can solve, simulate and estimate DSGE models. It is developed by the New Area-Wide Model (NAWM) team at the Monetary Policy Research Division (formerly the Econometric Modelling Division) within the Directorate General Research of the ECB. <http://www.texlips.net/yada/>

## II.1. What is Dynare?

- Alternatives to Dynare:
  - PyMacLab: Python Macroeconomics Laboratory, is a Python library to solve DSGE model. Developed by Erich Scheffel. Solution method similar to that of Dynare. <http://pypi.python.org/pypi/pymaclab>
  - SolveDSGE for Julia. Developed by Richard Dennis. This package can solve DSGE models and optimal policy problems. Solution methods include Blanchard-Kahn, Klein, Sims and Binder-Pesaran.

## II.1. What is Dynare?

- MMDB: Volker Wieland's Macroeconomic Model Database. This is a Matlab program that use Dynare to solve a large number of DSGE models.
- 14 Calibrated DSGE models.
- 26 Estimated DSGE models for the US.
- 9 Estimated euro area DSGE models, including the QUEST III model.
- 7 Calibrated/Estimated DSGE multicountry models.
- 5 Estimated DSGE models for other countries.



## II.1. What is Dynare?

- International Network for DSGE modeling, fiscal and monetary policy
- [www.dsge.net](http://www.dsge.net)
- DSGE tools and numerical tools. Including the Anderson-Moore algorithm (AIM), the Uhlig's toolkit, and several other Matlab programs.

## II.1. What is Dynare?

- Dynare is probably the most widely used computer application for solving and simulating DSGE models, both in the Academy and in economic institutions as central banks, ministries of economy and finance, international organizations and private financial institutions.
- This is because is very easy to work with. Very simple language and based on Matlab. Little (if any) programming skills in Matlab.
- Dynare cannot write a paper for you but almost!!!! In fact, you don't need to write the equations of the model in Latex, Dynare can do it for you.

## II.1. What is Dynare?

- Main advantage: Very little programming skills is required.  
Furthermore, close to zero Matlab programming skills required.
- Main disadvantage: Dynare is a black box.
- Learning by doing process.

## II.2. Installing and running Dynare

- Dynare website: <http://www.dynare.org>
- In the main menu go to Download. Then select Dynare stable (recommended).
- Last version is 4.4.3. Download this version (\*.exe file) and open the Dynare installation setup. By default, Dynare will be installed in the directory: `c:\dynare\4.4.3`
- Start Matlab. One option: Type directly in the Matlab command windows: `addpath c:\dynare\4.4.3\matlab`
- The second option is: Click in the File menu on Set Path. Select Add Folder...(Do not use Add with Subfolders). Select the directory where Dynare is installed and the subfolder matlab. Then select Save.  
DONE!!!!!!

## II.2. Installing and running Dynare

- To run Dynare in Matlab go to the command window.
- In order to give instructions to Dynare, we have to write a text file with extension \*.mod. This file will contains the definition of variables and parameters, the equations of the model and the required computing tasks. (Some computers consider the mod file as a video file and Windows Media will try to open it).
- Change the current folder to the one in which you have the Dynare program: A \*.mod file (example: `filename.mod`).
- Just type the command in the Matlab command window: `dynare filename`
- GOOD LUCK!!!!!!!

## II.2. Installing and running Dynare

- Dynare: Reference Manual Version 4. Adjemian et al. (2014)
- Dynare User Guide. Mancini (2008)

## II.2. Installing and running Dynare

- Basic RBC model. Very simple model economy setup:
  - Two type of economic agents: Households and firms.
  - Each agent maximizes a given objective function subject to a given restriction.
  - A lot of implicit assumptions: perfect capital markets, utility function additively separable in time, separability between consumption and leisure, saving as a state variable, perfect competition, constant return to scale, no externalities, etc. These implicit assumptions can be relaxed resulting in a large variety of DSGE models.

## II.3. Basics of the mod file

- To work with Dynare we must create first a file `*.mod`. This is an ordinary text file and it can be created with any text editor.
- Then, we write `dynare filename` in the Matlab command window.
- Dynare will produce Matlab programs `*.m` and other files: `*.log`, `*.eps`, `*.asv`;
- A mod file contains a list of commands and blocks. Important: Each command and each element of a block must be terminated by a semicolon (;). The meaning of ; in Dynare is different from Matlab.
- To comment a line Dynare use the symbol `//` (but you can also use the `%` of Matlab). We can also comment a set of lines with `/*` and `*/`
- Blocks start with the name of the block and are terminated by the command `end`;



## II.3. Basics of the mod file

- Standard structure of a Dynare program:
  - Definition of the endogenous variables
  - Definition of the exogenous variables
  - Definition of the parameters
  - Initial values (calibration) of the parameters
  - Equations of the model
  - Computing the steady state
  - Definition of shocks
  - Simulation and/or estimation of the model

## II.3. Basics of the mod file

- Standard structure of a Dynare program (Blocks):
  - Model Block
  - Initial values block
  - Final values block
  - Shocks Block

## II.3. Basics of the mod file

- Let's start looking to the basic structure of a Dynare file \*.mod for the following basic DSGE model (a system of 8 equations for 8 endogenous variables):

$$\begin{aligned}(1 - \gamma) \frac{1}{1 - L_t} &= \gamma \frac{1}{C_t} W_t \\ E_t \frac{C_{t+1}}{C_t} &= \beta E_t [R_{t+1} + 1 - \delta] \\ Y_t &= C_t + I_t \\ Y_t &= A_t K_t^\alpha L_t^{1-\alpha}\end{aligned}$$

## II.3. Basics of the mod file

- The other four equations are:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$W_t = (1 - \alpha)A_t K_t^\alpha L_t^{-\alpha}$$

$$R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$$

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A$$

## 11.3. Basics of the mod file

- Command var: Definition of endogenous variables.
- `var Y, C, I, K, L, W, R, A;`
- You can also write `var Y C I K L W R A;`
- You can use the command var as many times as you want. For instance:
  - `var Y C;`
  - `var I, K;`
  - `.....`
  - `var L W;`
- If you want to give a Latex name (for creating a tex file of the model) to the variables write: `var Y $Y$;`
- You can assign a long version of the variable name using the command (`long_name='variablename'`):
  - `var Y $Y$ (long_name='Output');`

## II.3. Basics of the mod file

- Command varexo: Definition of the exogenous variables.
- `varexo e;`
- You can use the command varexo as many times as you want.
- Giving a Latex name: `varexo e $\epsilon$;`
- In the case of a deterministic exogenous variable it can be used the command `varexo_det`. For instance, `varexo_det tauc;`

## II.3. Basics of the mod file

- Definition and values for the parameters:
  - `// Parameters names`
  - `parameters alpha, beta, delta, gamma, rho;`
  - `// Parameters values`
  - `alpha = 0.35;`
  - `beta = 0.97;`
  - `delta = 0.06;`
  - `gamma = 0.40;`
  - `rho = 0.95;`

## II.3. Basics of the mod file

- From National Accounts we know that  $\alpha$  is around 0.35. Also from econometric analyses. Estimated values of capital income shares for different countries can be found, for instance, in Gollin (2002).<sup>1</sup>
- Utility discount factor,  $\beta$ , takes values around 0.97 for annual data and around 0.99 for quarterly data.
- Physical capital depreciation rate,  $\delta$ , are obtained from capital stock database, and value are around 6% yearly or 2,5% quarterly.
- The preference parameter  $\gamma$  can be calculated as the ratio of working hours to total available time.
- Autorregresive parameter for TFP is typically sets at 0.95.

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<sup>1</sup>Gollin, D. (2002): Getting income shares right. *Journal of Political Economy*, 110, 458-474.



## II.3. Basics of the mod file

- The equations of the models must be defined in the model block. This block starts with the command `model`; (then the equations), and end with the command `end`;
- Time subscripts must be introduced as follows:
  - $X_t \implies X$
  - $X_{t+1} \implies X(+1)$
  - $X_{t-1} \implies X(-1)$
- All variables known at time  $t$  must be dated as time  $t - 1$ . For instance,  $K_t$  is the capital stock to be chosen in period  $t$ , whereas  $K_{t-1}$  is the capital stock available at beginning of period  $t$ .
- A variable  $(+1)$  in a equation indicates Dynare that the equation has conditional expectations.

## II.3. Basics of the mod file

- Equations of the model:

- `// Model Block`
- `model;`
- `C = (gamma/(1-gamma))*(1-L)*(1-alpha)*Y/L;`
- `1 = beta*((C/C(+1))*(R(+1)+(1-delta)));`
- `Y = A*(K(-1)^alpha)*(L^(1-alpha));`
- `K = (1-delta)*K(-1)+I;`
- `I = Y-C;`
- `W = (1-alpha)*A*(K(-1)^alpha)*(L^(-alpha));`
- `R = alpha*A*(K(-1)^(alpha-1))*(L^(1-alpha));`
- `log(A) = rho*log(A(-1))+ e;`
- `end;`

## II.3. Basics of the mod file

- What we need to have the model for Dynare is to solve by hand for the first order conditions. In this example we have five first order conditions (that will be reduced to four equations once we solve for the Lagrangian multiplier, the shadow price of consumption):
  - Labor supply
  - Consumption
  - Wages
  - Interest rate
- We have two additional static equations:
  - Output
  - Investment
- The law of motion for physical capital,
- And the stochastic process for the total factor productivity.

## II.3. Basics of the mod file

- Model equations can be written in Dynare with three different forms:
  - Equations is level
  - In exponential-log form
  - In linear form (first order approximation)
- Dynare uses Schmitt-Grohe and Uribe (2004) to compute first and second order approximation of the model.<sup>2</sup>

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<sup>2</sup>Schmitt-Grohe, S. and Uribe, M. (2004: Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control*, 28, 755-775.

## II.3. Basics of the mod file

- Initial value block: We must provide Dynare with initial values for the steady state computation.
  - `// Initial values`
  - `initval;`
  - `Y = 1;`
  - `C = 0.8;`
  - `L = 0.3;`
  - `K = 3.5;`
  - `I = 0.2;`
  - `W = (1-alpha)*Y/L;`
  - `R = alpha*Y/K;`
  - `A = 1;`
  - `e = 0;`
  - `end;`

## II.3. Basics of the mod file

- Initial values are settled as follows:
  - Consumption and investment are constant proportions of output. In our simple model, we can expect consumption to be around 80% of total output and investment the rest 20%.
  - Working hours are expected to be around 1/3 of total available time ( $\simeq 35$ -40 hours by week over  $16 \times 6 = 96$  hours).
  - Given  $\delta = 0.06$ , capital stock is expected to be  $0.2 \times Y / 0.06$ , that is, around 3.3 times output.
  - Given  $\alpha = 0.35$  and  $A = 1$  total output is

$$Y = (3.3 \times Y)^{0.35} 0.3^{0.65} = (1.518 \times 0.457)^{1/0.65} = 0.57$$

## II.3. Basics of the mod file

- Computing the steady state with Dynare. This is a very important and compulsory step in solving the DSGE model. If you want to know the steady state values for the endogenous variables, simply use the command:
  - `steady;`
- Independently of this command, Dynare will write always the static version of the model, just by dropping the time index for each variable and by setting  $\sigma_A = 0$ .
- This is the most difficult part for Dynare, given that it has to resolve a non-linear problem. Importance of very good initial conditions. Dynare uses `initval` as the guess for the steady state.

## II.3. Basics of the mod file

- Checking the uniqueness of the steady state. Blanchard-Khan (1980) rank condition<sup>3</sup>
- To compute the eigenvalues of the model:
  - check;
- Dynamic stability of the system. Necessary rank condition: The number of eigenvalues outside the unit circle must be equal to the number of forward-looking variables.

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<sup>3</sup>Blanchard, O.J. and Kahn, C.M. (1980): The solution of linear difference models under rational expectations. *Econometrica*, 48(5), 1305-1212.



## II.3. Basics of the mod file

- (Random) shock analysis. In the case of stochastic shocks, we must use the block shocks. This block start with the command `shocks;` and ends with the command `end;`
- In this block we must define the shock or shocks to be simulated, the variances and covariances:
  - `// Shocks Block`
  - `shocks;`
  - `var e;`
  - `stderr 0.01;`
  - `end;`
- The covariance between two shocks can be defined as:
  - `var e1 e2 = 0.001;`

## II.3. Basics of the mod file

- The last step consists in simulating the model. Dynare can do stochastic or deterministic simulations.
- For stochastic simulation: Command `stoch_simul`;
- For deterministic simulation: Command `simul`;
- A large number of options

## II.3. Basics of the mod file

- Basic DSGE model.
- File example1.mod
- Total Factor Productivity shock.
- First order approximation.
- Just type `dynare example1` in the Matlab command window.

## II.4. The output

- In the case of a stochastic simulation we get in the Matlab command window the following information:
  - STEADY-STATE RESULTS
  - EIGENVALUES (rank condition)
  - MODEL SUMMARY (number of variables, number of stochastic shocks, number of state variables, number of jumpers and number of static variables)
  - MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS
  - POLICY AND TRANSITION FUNCTIONS
  - THEORETICAL MOMENTS (mean, standard deviation and variance)
  - MATRIX OF CORRELATION
  - COEFFICIENT OF AUTOCORRELATION
  - GRAPHS: IMPULSE RESPONSE FUNCTIONS

## 11.4. The output

- Dynare prints everything by default.
- You can use some options to cancel output from the simulation prints:
  - `noprnt`: cancel any printing
  - `nocorr`: cancel the print of the correlation matrix
  - `nofunctions`: cancel the print of the approximated solutions
  - `nomoments`: cancel the print of the moments of endogenous variables
  - `irf=0`: cancel the print of the impulse-response graphs

## II.4. The output

- A DSGE model can be represented as a system of stochastic equations:

$$E_t \{f(y_{t+1}, y_t, y_{t-1}, x_t, \varepsilon_t)\} = 0$$

$\varepsilon$ : stochastic shock.  $E_t(\varepsilon_t) = 0, E_t(\varepsilon_t \varepsilon_t') = \Sigma$ .

- Solving the model implies finding an unknown decision function:

$$y_t = g(x_{t-1}, \varepsilon_t)$$

that can be introduced into the model and that satisfy the first order conditions and feasibility conditions.

## 11.4. The output

- The first order approximation to the decision functions can be written as:

$$y_t = \bar{y} + g_{y,x}(x_{t-1} - \bar{x}) + g_u \varepsilon_t$$

where  $g_x$  is the partial derivative of the function  $g$  with respect to the variable  $x$ .

- The function  $g(\cdot)$  is an approximated recursive representation of the model that can be used to generate time series of the endogenous variables that satisfy the rational expectations hypothesis of the model.

## 11.4. The output

- **Steady state:**
- Computing the steady state:

$$f(\bar{y}, \bar{y}, \bar{y}, \bar{x}, 0) = 0$$

$$\bar{y} = g(\bar{y}, \bar{x}, 0)$$

- Using of the Taylor expansion function on the structural model in order to compute the coefficient of the unknown decision function.



## II.4. The output

- Steady State: To compute the steady state, first, eliminate time. Second, assume that  $\sigma_A = 0$ . This means, for instance, that we would have an equilibrium value for consumption such as  $\dots = C_{t-1} = C_t = C_{t+1} = \dots = \bar{C}$ . Thus, the equations of the model can be written as:

$$\frac{(1-\gamma)}{\gamma} \frac{\bar{C}}{1-\bar{L}} = (1-\alpha) \bar{A} \bar{K}^\alpha \bar{L}^{-\alpha} \quad (1)$$

$$1 = \beta [\bar{R} + 1 - \delta] \quad (2)$$

$$\bar{Y} = \bar{A} \bar{K}^\alpha \bar{L}^{1-\alpha} \quad (3)$$

$$\bar{I} = \delta \bar{K} \quad (4)$$

$$\bar{C} + \bar{I} = \bar{Y} \quad (5)$$

## 11.4. The output

- From equation (2) it can be obtained the steady state value for the rental rate of capital:

$$\bar{R} = \frac{1}{\beta} + \delta - 1$$

- Note that:

$$\bar{R} = \alpha \frac{\bar{Y}}{\bar{K}}$$

## 11.4. The output

- The easy thing is to first express all the steady state values in terms of the steady state value for output. Expression (2) can be written as:

$$1 = \beta \left[ \alpha \frac{\bar{Y}}{\bar{K}} + 1 - \delta \right]$$

- Solving for  $\bar{K}$  results:

$$\bar{K} = \frac{\alpha\beta}{1 - \beta + \beta\delta} \bar{Y} \quad (6)$$

## 11.4. The output

- Second, using (4) and given (6) the investment steady state value is given by:

$$\bar{I} = \frac{\alpha\beta\delta}{1 - \beta + \beta\delta} \bar{Y} \quad (7)$$

- Third, using expressions (5) and (7) we reach the steady state value for consumption:

$$\bar{C} = \frac{1 - \beta + (1 - \alpha)\beta\delta}{1 - \beta + \beta\delta} \bar{Y} \quad (8)$$

## 11.4. The output

- Next, using (1) we obtain:

$$\bar{L} = \frac{\gamma(1-\alpha)(1-\beta+\beta\delta)}{(1-\gamma)(1-\beta+(1-\alpha)\beta\delta) + \gamma(1-\alpha)(1-\beta+\beta\delta)} \quad (9)$$

- Finally, substituting (6) and (9) in (3) we reach the steady state value of output:

$$\bar{Y} = \bar{A}^{\frac{1}{1-\alpha}} \left[ \frac{\alpha\beta}{1-\beta+\beta\delta} \right]^{\frac{\alpha}{1-\alpha}} \quad (10)$$

$$\left[ \frac{\gamma(1-\alpha)(1-\beta+\beta\delta)}{(1-\gamma)(1-\beta+(1-\alpha)\beta\delta) + \gamma(1-\alpha)(1-\beta+\beta\delta)} \right] \quad (11)$$

## II.4. The output

- Using the command `steady`; we obtain the following output:

STEADY-STATE RESULTS:

Y	0.744697
C	0.572708
I	0.17199
K	2.86649
L	0.360396
W	1.34312
R	0.0909278
A	1

## II.4. The output

- The calculated steady state value for output is:

$$\bar{Y} = \bar{A}^{\frac{1}{1-\alpha}} \left[ \frac{\alpha\beta}{1-\beta+\beta\delta} \right]^{\frac{\alpha}{1-\alpha}} \left[ \frac{\gamma(1-\alpha)(1-\beta+\beta\delta)}{(1-\gamma)(1-\beta+(1-\alpha)\beta\delta) + \gamma(1-\alpha)(1-\beta+\beta\delta)} \right]$$

given that  $\alpha = 0.35, \beta = 0.97, \delta = 0.06, \gamma = 0.4$ , the resulting number is:

$$\bar{Y} = 2.066330915 \times 0.36039604 = 0.744697$$

- We can recover the rest of steady state values. For instance:

$$\bar{K} = \frac{\alpha\beta}{1-\beta+\beta\delta} \bar{Y} = 3.8492 \times \bar{Y} = 2.8664$$

## II.4. The output

- Uniqueness of the steady state. Using the command `check`; we obtain the following output:

EIGENVALUES:

Modulus	Real	Imaginary
0.8866	0.8866	0
0.95	0.95	0
1.163	1.163	0
Inf	Inf	0

There are 2 eigenvalue(s) larger than 1 in modulus  
for 2 forward-looking variable(s)

The rank condition is verified.



## II.4. The output

- This is the so-called Blanchard and Kahn condition.<sup>4</sup> This is the first solution method proposed for DSGE models.
- Number of eigenvalues outside the unitary circle must be equals to the number of forward-looking variables.
- If the rank condition is not satisfied, then the steady state does not exist or there are infinity paths to the steady state.

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<sup>4</sup>Blanchard and Kahn (1980).

## 11.4. The output

- Briefly, the rank condition can be defined as follows. First, we can write the following systems:

$$Ay_t = Bx_t + C\hat{a}_t \quad (12)$$

$$DE_tx_{t+1} + FE_ty_{t+1} = Gx_t + Hy_t \quad (13)$$

- The matrix model is closed by incorporating the expected deviation of total factor productivity:

$$E_t\hat{a}_{t+j} = \rho_A^j \hat{a}_t$$

where  $y_t$  is the vector of deviations from the steady state for the endogenous variables (not predetermined or jumps) and  $x_t$  is the vector of deviations from the steady state for the predetermined and jumps variables.

## II.4. The output

- The system (12) can be written as:

$$y_t = A^{-1}Bx_t + A^{-1}C\hat{a}_t$$

- Taking one period ahead, the above system should be:

$$E_t y_{t+1} = A^{-1}B E_t x_{t+1} + A^{-1}C \rho_A \hat{a}_t$$

- Substituting in the system (13) we find that:

$$(D + FA^{-1}B)E_t x_{t+1} = (G + HA^{-1}B)x_t + (HA^{-1}C - FA^{-1}C\rho_A)\hat{a}_t$$

## II.4. The output

- Solving for the matrices, the final system would be:

$$E_t x_{t+1} = Jx_t + M\hat{a}_t$$

where:

$$J = (D + FA^{-1}B)^{-1}(G + HA^{-1}B)$$

$$M = (D + FA^{-1}B)^{-1}(HA^{-1}C - FA^{-1}C\rho_A)$$

## 11.4. The output

- Using the Jordan decomposition, the matrix  $J$  can be decomposed such as:

$$J = O^{-1}NO$$

where:

$$N = \begin{bmatrix} N_{11} & 0 \\ 0 & N_{22} \end{bmatrix}$$

and where:

$$O = \begin{bmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{bmatrix}$$

- The elements of the diagonal of  $N$  are the eigenvalues of the matrix  $J$ . In order the solution to be unique, the value of  $N_{11}$  must be inside the unit circle and the value of  $N_{22}$  outside the unit circle. This is the so-called the Blanchard-Kahn rank condition. If the rank condition does not hold, then the equilibrium is not unique. The columns of  $O^{-1}$  are the eigenvectors of the matrix  $J$ .

## II.4. The output

- Too many eigenvalues larger than one: No stable solution.
- Not enough eigenvalues larger than one: Multiple equilibria.
- Number of eigenvalues larger than one equals to the number of forward-looking variables: Uniqueness.

## II.4. The output

- If the `noprint` option is not used, then we obtain a summary of the number of variables of the model classified for types:

### MODEL SUMMARY

Number of variables:	8
Number of stochastic shocks:	1
Number of state variables:	2
Number of jumpers:	2
Number of static variables:	4

## II.4. The output

- In our example model, output, investment, labor, and wage are static variables.
- Capital stock and TFP are purely predetermined (state) variables.
- Jumping variables (forward-looking, control variables).
- Internally, Dynare sorts the variables in the following order:
  - Static variables
  - Purely predetermined variables (variables appearing in time  $t$  and  $t - 1$ )
  - Predetermined and forward-looking variables (variables appearing in  $t + 1$ ,  $t$  and  $t - 1$ )
  - Purely forward-looking variables (variables appearing in  $t + 1$  and  $t$ )



## II.4. The output

- Variance-covariance matrix for the exogenous shocks:

MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS

Variables    e

e            0.000100

## 11.4. The output

- **Policy and transition functions:** The solution for a DSGE model consists in the description of the optimal plan by finding a rule that tells what the control should be, given any possible value of the states. The rules determining the controls as a function of the states is what is called a policy function. The rules determining the states as a function of the states is what is called a transition function.
  - $y_t$ : control
  - $x_t$ : state
  - $a_t$ : exogenous stochastic process

$$x_t = Px_{t-1} + Qa_t$$

$$y_t = Rx_{t-1} + Sa_t$$

where  $P$ ,  $Q$ ,  $R$  and  $S$  are matrices such that the computed equilibrium is stable.

## III.4. The output

- **Policy and transitions functions** (this output can be cancelled with the option `nofunctions`):

POLICY AND TRANSITION FUNCTIONS								
	Y	C	I	K	L	W	R	A
Constant	0.744697	0.572708	0.171990	2.866494	0.360396	1.343115	0.090928	1.000000
K(-1)	0.055192	0.108550	-0.053358	0.886642	-0.026607	0.198700	-0.024982	0
A(-1)	0.923502	0.310584	0.612918	0.612918	0.160849	1.066152	0.112760	0.950000
e	0.972107	0.326930	0.645177	0.645177	0.169315	1.122265	0.118695	1.000000

- This is the final solution of the model. Dynare use the Klein (2000) method.<sup>5</sup>

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<sup>5</sup>Klein P. (2000): Using the generalized Schur form to solve a multivariate inear rational expectations model. *Journal of Economic Dynamic and Control*, 24(1), 405-423.

## II.4. The output

- For instance, the policy function computed by Dynare for output is:

$$Y_t = 0.744 + 0.055 \times (K_{t-1} - \bar{K}) + 0.923 \times (a_{t-1} - \bar{a}) + 0.972 \times \varepsilon_t$$

- Writing policy functions in this way is a peculiarity of Dynare. In general policy functions are written as follows:

$$Y_t = 0.744 + 0.055 \times (K_{t-1} - \bar{K}) + 0.972 \times (a_t - \bar{a})$$

$$a_t = \rho a_{t-1} + \varepsilon_t$$

- Note that 0.923 is just 0.972 times 0.95.

## 11.4. The output

- Theoretical moments (this output can be cancelled with the option `nomoments`):

THEORETICAL MOMENTS			
VARIABLE	MEAN	STD. DEV.	VARIANCE
Y	0.7447	0.0383	0.0015
C	0.5727	0.0258	0.0007
I	0.1720	0.0148	0.0002
K	2.8665	0.1527	0.0233
L	0.3604	0.0032	0.0000
W	1.3431	0.0632	0.0040
R	0.0909	0.0023	0.0000
A	1.0000	0.0320	0.0010

## 11.4. The output

- Correlation matrix (this output can be cancelled with the option `nocorr`):

### MATRIX OF CORRELATIONS

Variables	Y	C	I	K	L	W	R	A
Y	1.0000	0.9681	0.8993	0.9165	0.5672	0.9881	0.1745	0.9918
C	0.9681	1.0000	0.7612	0.9875	0.3429	0.9951	-0.0776	0.9283
I	0.8993	0.7612	1.0000	0.6492	0.8702	0.8215	0.5875	0.9477
K	0.9165	0.9875	0.6492	1.0000	0.1903	0.9670	-0.2340	0.8580
L	0.5672	0.3429	0.8702	0.1903	1.0000	0.4340	0.9099	0.6676
W	0.9881	0.9951	0.8215	0.9670	0.4340	1.0000	0.0212	0.9605
R	0.1745	-0.0776	0.5875	-0.2340	0.9099	0.0212	1.0000	0.2986
A	0.9918	0.9283	0.9477	0.8580	0.6676	0.9605	0.2986	1.0000

## 11.4. The output

- Correlation matrix (this output can be cancelled with the option `nocorr`):

### COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
Y	0.9672	0.9341	0.9010	0.8679	0.8352
C	0.9911	0.9779	0.9613	0.9419	0.9202
I	0.8999	0.8104	0.7305	0.6591	0.5951
K	0.9969	0.9887	0.9761	0.9600	0.9410
L	0.8431	0.7062	0.5868	0.4830	0.3928
W	0.9837	0.9644	0.9426	0.9190	0.8939
R	0.8511	0.7209	0.6072	0.5079	0.4214
A	0.9500	0.9025	0.8574	0.8145	0.7738

## 11.4. The output

- **Impulse response functions:**
- `irf=INTEGER`; Number of periods to compute the IRFs.
  - `INTEGER=0`; Does not compute the IRF.
  - `relative_irf`; Compute the normalized IRF in percentage of the standard error of each shock.
- Default: Dynare computes and plots the IRF for 40 periods. If quarterly data (10 years). If annual data (40 years).



## 11.4. The output

- Where the output is stored?: In several string memory allocations.
- `oo_` : Main information about model results.
- `M_` : Structure containing information about the model
- `options_` : Information about the options used in the computation.
- `estimation_info` : Information about the model estimation results.
- For instance, if we want to recover the steady state values of the variables we type: `oo_.steady_state`

## II.4. The output

- Output is saved in the file `namefile.log`
- The variables and matrices created when solving the model are stored in the file `filename_results.mat`
- Figures are saved in `*.eps`, `*.fig`, and `*.pdf` formats

## II.5. More on the mod file

- You can combine Dynare commands with Matlab commands in the same mod file.
- Be careful because Dynare clear the memory before starting, and then previous Matlab calculations are deleted.
- To avoid that use the option [noclearall]: `dynare namefile noclearall`
- File example2.mod. RBC basic analysis.

## II.5. More on the mod file

- Dynare can write for you a Latex file with the equations of the model. This is very useful for checking for errors, specially for large models. The most important error in DSGE modelling is the "non error" (specially for large scale models).
- If you want to give a Latex name (for creating a tex file of the model) to the variables write: `var Y $Y$;`
- Giving a Latex name to the exogenous variables: `varexo e $\epsilon$;`
- Giving a Latex name to the parameters: `parameters alpha $\alpha$;`
- To obtain a Latex file with the static model:  
`write_latex_static_model;`
- To obtain a Latex file with the dynamics model:  
`write_latex_dynamic_model;`
- File `example3.mod`

## 11.5. More on the mod file

- **Timing in Dynare:** This is important. For stock variables, Dynare uses a "stock at the end of the period" concept, instead of the standard "stock at the beginning of the period" concept.
- The typical example is how we define the physical capital stock accumulation equation. In a theoretical DSGE model we write:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- However, in Dynare that equation must be written as:

$$K = (1 - \text{delta}) * K(-1) + I$$

- That is, the capital stock used at current period is actually decided in the previous period. Investment cannot just disappearing every period and then reappearing in the following period.

## II.5. More on the mod file

- **Timing in Dynare:** This is important. For stock variables, Dynare uses a "stock at the end of the period" concept, instead of the standard "stock at the beginning of the period" concept.
- Production function also must be changed. For instance, in a theoretical DSGE model we write:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

- However, in Dynare that equation must be written as:

$$Y = A * K(-1)^\alpha * L^{(1 - \alpha)}$$

- That is, the capital stock used at current period is stock of capital accumulated in the previous period.

## II.5. More on the mod file

- That is, theoretically we have:

$$(1 - \gamma) \frac{1}{1 - L_t} = \gamma \frac{1}{C_t} W_t$$

$$\frac{C_t}{C_{t-1}} = \beta [R_t + 1 - \delta]$$

$$Y_t = C_t + I_t$$

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

$$W_t = (1 - \alpha) A_t K_t^\alpha L_t^{-\alpha}$$

$$R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$$

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A$$

## II.5. More on the mod file

- But, for Dynare the model is:

$$(1 - \gamma) \frac{1}{1 - L_t} = \gamma \frac{1}{C_t} W_t$$

$$\frac{C_{t+1}}{C_t} = \beta [R_{t+1} + 1 - \delta]$$

$$Y_t = C_t + I_t$$

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha}$$

$$K_t = (1 - \delta) K_{t-1} + I_t$$

$$W_t = (1 - \alpha) A_t K_{t-1}^\alpha L_t^{-\alpha}$$

$$R_t = \alpha A_t K_{t-1}^{\alpha-1} L_t^{1-\alpha}$$

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A$$



## II.5. More on the mod file

- **predetermined\_variables**: It is possible to change the above interpretation for the stock variables.
- File example4.mod

## 11.5. More on the mod file

- **change\_type**: Example: `change_type(var) alpha, beta;`  
`change_type(parameters) Y, C;`
- Command for changing the type of the specified endogenous/exogenous variables or parameters to another type.
- `change_type(var); change_type(parameter);`  
`change_type(varexo); change_type(varexo_det);`
- This allows us to do inverse calibration, i.e., using data and equilibrium conditions from the DSGE model to calibrate some parameters.
- File `example5.mod`

## II.5. More on the mod file

- The model can be nonstationary. In this case Dynare need the trend deflator for the nonstationary variables in order to stationarize the model.
- **trend\_var**: This is the command for declaring trend variables.
  - `trend_var (growth_factor=gA) A;`
  - `log_trend_var (log_growth_factor=ga) a;`
- The `trend_var` command must appear before the `var` command for the trend variable.
- `[deflator=name]`: This expression is used to detrend nonstationary endogenous variables.
- `[log_deflator=name]`: For additive trend.
- File example6.mod

## II.5. More on the mod file

- Model block options. `model (OPTIONS,...);`
- `model (linear);` To indicate Dynare that the model is linear. This reduces the time of computation.
- `model (no_static);` To indicate Dynare not to create the static version of the model.
- `#` inside the model block: To create model local variables. For instance:
  - `model;`
  - `...;`
  - `# sigma=1+1/eta;`
  - `...`
  - `end;`

## II.5. More on the mod file

- `model (block)`; To indicate Dynare to perform a block decomposition of the model. This will be used for steady state computation, deterministic simulation and stochastic simulation with a first order approximation. Option needed to use some versions of the Newton type algorithm.
- `model (bytecode)`; To indicate Dynare to write a binary file with a compact representation of all the equations of the model instead of writing \*.m files. Option needed to use some versions of the Newton type algorithm.

## 11.5. More on the mod file

- Dynare does linear approximations of the level of the variables. However, we typically prefer approximation to the natural logs of the variables (not all DSGE people think that) so impulse responses are given in percentage terms.
- If we want Dynare to do log-linearization rather than linearization, then we must change model equations by taking exponential of all variables.
- To say Dynare to do linear approximations of the logs of the variables, we only need to specify the variables of the model as  $\exp(x)$ . Doing that, Dynare interprets the variable  $x$  as the log of the variable of interest, while  $\exp(x)$  is interpreted as the level of the variable.
- File example7.mod

## II.5. More on the mod file

- Relationship between `steady` and the block `initval`. If `initval` block is not followed by the command `steady`, both stochastic simulation and estimation will compute the steady state. Deterministic simulation will use the `initval` values as the starting values for endogenous variables instead of steady state values.
- `initval`: numerical initial guess values for the non-linear solver. Given these initial values Dynare will start numerically searching for the steady state.
- Command `resid`; Used to compute equation residuals for the initial values.
- If a variable is not included in the `initval` block, a zero value is assumed.

## II.5. More on the mod file

- Block `endval`. Only for the case of deterministic models.
- This block provides terminal conditions for all the periods succeeding the last simulation period.
- Second, this block provides initial guess values for the non-linear solver implemented in the command `simul`.
- If the `endval` block is followed by the command `steady`, Dynare uses the values for the endogenous variables of the `endval` block as initial guess values for the non-linear solver, given the exogenous variables values in the `endval` block.
- If an `endval` block is not followed by a `shocks` block, Dynare assumes that the variables move to the values in `endval` and stay there forever.



## II.5. More on the mod file

- DGE versus DSGE models: The distinction between a deterministic and a stochastic model is related to whether future shocks are known.
- In a deterministic model, all future shocks are known at the time when agents take their decisions.
- In a stochastic model, only the distribution of all future shocks is known at the time agents take their decisions.

## II.5. More on the mod file

- Simulation of endogenous variables. Command `periods=INTEGER;`
- `periods=1000;` This can be included in any line or in the `stoch_simul` command as an option.
- File `example8.mod`
- Command `rplot;`

## II.5. More on the mod file

- Random number generator. In Dynare the random number generator seed has been fixed by default to generate results that are replicable across different runs on different computers at different times, i.e., the random number generator is deterministic. Dynare generates random number algorithm with a seed equals to 0.
- If you want to generate several different simulation samples, you should use the command `set_dynare_seed('clock')`; where the seed had been set to the computer clock.

## II.5. More on the mod file

- `Extended_path`; Dynare can also simulate a stochastic model using the extended path method of Fair and Taylor (1983).<sup>6</sup> Recommended when there are strong nonlinearities or binding constraints.
- This method computes a random path for the exogenous variables and then computes the corresponding path for endogenous variables, assuming that the agents believe that there will no more shocks in the following periods.

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<sup>6</sup>Fair, R. and Taylor, J. (1983): Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models. *Econometrica*, 51, 1169-1185.

## II.5. More on the mod file

- `hp_filter=INTEGER`; This produces theoretical moments after HP filtering the data.
- `INTEGER=128000` for monthly data, 1600 for quarterly data and 6.25 for annual data.
- File `example9.mod`

## 11.6. Deterministic simulation

- To solve a deterministic model (DGE), Dynare uses the relaxation method.
- `simul (OPTIONS...);`
- Deterministic models (DGE) assume perfect foresight.
- In general, it is assumed that the initial point is the steady state.
- Dynare can simulate how the economy responds to the news of a contemporaneous shock (non-anticipated) or a future shock (anticipated).
- The dynamic of the economy can be toward a new steady state (permanent shock) or to returns to the initial steady state (transitory shock).
- In this case the numerical problem consists in solving a non-linear system of simultaneous equations for a set of endogenous variables in  $T$  periods.

## II.6. Deterministic simulation

- Deterministic GE models are used to study the impact of a regime change (for instance, the introduction of a new tax).
- No uncertainty about shocks, that is, future shocks are known. Perfect foresight.
- Dynare solves the model without linearization.
- In this case, the economy can be far away from the steady state.

## II.6. Deterministic simulation

- `simul (periods=40)`; Number of period of the simulation.
- `simul (maxit=20)`; Maximum number of iterations to be used by the non-linear solver (Default=10).
- `simul (stack_solve_algo=INTEGER)`; Algorithm to be used in the solution (Default=0).
  - `INTEGER=0`: Newton method using sparse matrices.
  - `INTEGER=1`: Newton method with sparse LU (requires `bytecode` or `block`).
  - `INTEGER=2`: Newton method with Generalized Minimal Residual (requires `bytecode` or `block`).
  - `INTEGER=3`: Newton method with Stabilized Bi-Conjugate Gradient (requires `bytecode` or `block`).
  - `INTEGER=4`: Newton method with an optimal path length (requires `bytecode` or `block`).
  - `INTEGER=5`: Newton method with sparse Gaussian elimination (requires `bytecode` or `block`).
  - `INTEGER=6`: The Julliard (1996) relaxation algorithm.



## II.6. Deterministic simulation

- DGE model with taxes. Three taxes: consumption tax, labor income tax and capital income tax.
- The only change in the basic model is in the households budget constraint:

$$(1 + \tau_t^c)C_t + K_t - K_{t-1} = (1 - \tau_t^l)W_tL_t + (1 - \tau_t^k)(R_t - \delta_K)K_{t-1} + Tr_t$$

where  $\tau_t^c, \tau_t^l, \tau_t^k$  are the consumption tax, the labor income tax and the capital income tax, respectively.  $Tr_t$  are lump-sum transfers.

## II.6. Deterministic simulation

- The government: It is assumed that the government uses tax revenues to finance lump-sum transfers paid out to the consumers. We assume that the government balances its budget period-by-period by returning revenues from distortionary taxes to the agents via lump-sum transfers,  $T_t$ .
- The government obtains resources from the economy by taxing consumption and income from labor and capital, whose effective average taxes are  $\tau_t^c$ ,  $\tau_t^l$ ,  $\tau_t^k$ , respectively.
- The government budget in each period is given by,

$$\tau_t^c C_t + \tau_t^l W_t L_t + \tau_t^k (R_t - \delta_K) K_t = G_t$$

- The government keeps a fiscal balance in each period  $G_t = T_t$ . This assumption is made to highlight the distortionary effects of taxes.

## II.6. Deterministic simulation

Model with taxes:

$$\begin{aligned}\frac{1}{1 - L_t} &= \frac{\gamma}{(1 - \gamma)} \frac{(1 - \tau_t^l)}{(1 + \tau_t^c)} \frac{W_t}{C_t} \\ \frac{(1 + \tau_t^c) C_t}{(1 + \tau_{t-1}^c) C_{t-1}} &= \beta \left[ (1 - \tau_t^k) (R_t - \delta) + 1 \right] \\ Y_t &= C_t + I_t \\ Y_t &= AK_t^\alpha L_t^{1-\alpha} \\ K_{t+1} &= (1 - \delta) K_t + I_t \\ W_t &= (1 - \alpha) AK_t^\alpha L_t^{-\alpha} \\ R_t &= \alpha AK_t^{\alpha-1} L_t^{1-\alpha}\end{aligned}$$

## II.6. Deterministic simulation

- Contemporaneous transitory tax change, i.e., a consumption tax increases during four periods.
- Example10.mod

```
//*****  
//   Block 6: Shocks  
//*****  
// Perturbation  
shocks;  
var tauc;  
// Periods in which the shock is in place  
periods 1:4;  
// Value of the shocks (new tax rate)  
values 0.15;  
end;
```

## II.6. Deterministic simulation

- Future anticipated transitory tax change, i.e., a consumption tax increases during four periods to occur in six periods.
- Example11.mod

```
//*****  
//   Block 6: Shocks  
//*****  
// Perturbation  
shocks;  
var tauc;  
  
// Periods in which the shock is in place  
periods 6:9;  
  
// Value of the shocks (new tax rate)  
values 0.15;  
end;
```

## II.6. Deterministic simulation

- We can also simulate different temporary shocks in different time periods. For instance:
  - `shocks;`
  - `var tauc;`
  - `periods 1:4 5:8;`
  - `values 0.15 0.18;`
  - `end;`
- Example12.mod

## II.6. Deterministic simulation

- Instantaneous permanent tax change, i.e., a consumption tax increases.
- Example13.mod

```
//*****  
//   Block 6: Shocks  
//*****  
// Perturbation  
shocks;  
var tauc;  
// Periods in which the shock is in place  
periods 0;  
// Value of the shocks  
values 0;  
end;
```

## II.6. Deterministic simulation

- Future permanent tax change, i.e., a consumption tax increases.
- Example14.mod

```
//*****  
//   Block 6: Shocks  
//*****  
// Perturbation  
shocks;  
var tauc;  
// Periods in which the shock is in place  
periods 1:10;  
// Value of the shocks  
values 0.12;  
end;
```



## II.7. Stochastic simulation

- In a DSGE shocks can only be instantaneous (non-anticipated) and transitory. A shock hits the economy today, but thereafter its expected value is zero.
- In this context, only the distribution of future shocks is known.
- We cannot study the effect of a permanent shock in a stochastic model.

## II.7. Stochastic simulation

- `stoch_simul (OPTIONS...) [variablename]`; This command solves a DSGE model with expectations using the perturbation method.
- With this command Dynare compute a Taylor approximation around the steady state for the transition and decision functions of the model.
- By default, Dynare use a second order Taylor approximation to solve DSGE models.
- Example 15.mod

## II.7. Stochastic simulation

- First order approximation (order=1):
  - Solution assumes that  $E_t(\varepsilon_{t+1}) = 0$
  - Other properties of the error distribution do not matter
- Second order approximation (order=2):
  - Solution assumes that  $E_t(\varepsilon_{t+1}) = 0$
  - $\sigma_\varepsilon$  matters as it affects the constant of the policy functions
  - Other properties of the error distribution do not matter

## II.7. Stochastic simulation

- Example16.mod. Investment-specific technological change (ISTC)

$$K_{t+1} = (1 - \delta)K_t + Q_t I_t$$

where  $Q_t$  represents ISTC. It is assumed that:

$$\log Q_t = \rho_Q \log Q_{t-1} + \eta_t$$

- When shocks are correlated the variance decomposition depends upon the order of the variables in the varexo command as they are computed using a Cholesky decomposition.
- Variance decompositions:  
conditional\_variance\_decomposition=[INTEGER1, INTEGER2]