Lecture 9: The consumption-leisure decision

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Advanced Macroeconomics

9. The consumption-leisure decision

- Extension of the basic consumer problem: Leisure as an additional argument in the utility function.
- Total available time (discretionary time) is split between leisure and working activities.
- Labor supply function.

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- Instantaneous utility function:

$$U(C_t, O_t) \tag{1}$$

- Assumptions: strictly increasing, strictly concave and two times differentiable:

$$U_C(\cdot) > 0 \qquad U_O(\cdot) > 0 \tag{2}$$

$$U_{CC}(\cdot) < 0 \qquad U_{OO}(\cdot) < 0 \tag{3}$$

- The cross derivative for consumption and leisure is positive:

$$U_{CO}(\cdot) > 0 \tag{4}$$

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- Household maximization problem is given by:

$$\max_{\{C_t, O_t\}_{t=0}^{T}} E_t \sum_{t=0}^{T} \beta^t U(C_t, O_t) \tag{5}$$

where E_t is the conditional expectation operator at time t, given the information available at that time and where β is the intertemporal discount factor, $\beta \in (0, 1)$, where:

$$\beta = \frac{1}{1+\theta} \tag{6}$$

and where θ is the rate of time preference $(\theta > 0)$.

- Budget constraint:

$$C_t + B_t = W_t L_t + (1 + R_t) B_{t-1} \tag{7}$$

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- Consumers choose an optimal consumption path for all their lifetime. They also choose the time devoted to working activities, period by period.
- Additional restriction:

$$O_t = 1 - L_t \tag{8}$$

where the total available discretionary time is normalized to 1.

- Household maximization problem with finite live and no uncertainty is given by:

$$\max_{\{C_t, L_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t U(C_t, 1 - L_t) \tag{9}$$

subject to:

$$\begin{aligned} C_t + B_t &= W_t L_t + (1 + R_t) B_{t-1} && (10) \\ B_{-1} &= 0 && (11) \\ B_T &= 0 && (12) \end{aligned}$$

Household problem	
Utility function	$U = U(C_t, O_t)$
Budget constraint	$C_t + B_t = (1 + R_{t-1})B_{t-1} + W_t L_t$
Initial stock of assets	$B_{-1} = 0$
Final stock of assets	$B_T = 0$
Time restriction	$L_t + O_t = 1$

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The auxiliary Lagrangian function to this problem is given by:

$$\mathcal{L} = \sum_{t=0}^{T} \left[\beta^t U(C_t, 1 - L_t) - \lambda_t (C_t + B_t - W_t L_t - (1 + R_t) B_{t-1}) \right] \quad (13)$$

where λ_t is the Lagrange multiplier. First order conditions for maximization are given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t U_C(C_t, 1 - L_t) - \lambda_t = 0 \quad (14)$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = \beta^t U_L(C_t, 1 - L_t) - \lambda_t = 0 \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} (1 + R_{t+1}) = 0 \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = C_t + B_t - W_t L_t - (1 + R_t) B_{t-1} = 0 \quad (17)$$

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- From the first order condition we have that,

$$\lambda_t = \beta^t U_C(C_t, 1 - L_t) \tag{18}$$

  where the Lagrange multiplier is the shadow price of consumption (marginal utility of consumption).

- Additionally, from the first order condition (15), the Lagrange multiplier can also be defined as the marginal disutility of labor::

$$\lambda_t = -\frac{\beta^t U_L(C_t, 1 - L_t)}{W_t} \tag{19}$$

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- Equating both expression, we have the equilibrium condition for which the marginal rate of substitution between consumption and leisure is equal to the oportunity cost of an additional unit of leisure:

$$U_C(C_t, 1 - L_t)W_t = -U_L(C_t, 1 - L_t) \tag{20}$$

  This equilibrium condition is the labor supply function.

- On the other hand, by substituting into (16) we have that:

$$\beta(1 + R_t)U_C(C_{t+1}, 1 - L_{t+1}) = U_C(C_t, 1 - L_t)$$

  that is, the equilibrium condition representing the optimal consumption path.

Household problem solution	
Optimal consumption path	$U_C(C_t, 1 - L_t) = \beta(1 + R_t)U_C(C_{t+1}, 1 - L_{t+1}$
Financial assets dynamics	$B_t = (1 + R_{t-1})B_{t-1} + W_t L_t - C_t$
Labor supply	$U_C(C_t, 1 - L_t)W_t = -U_L(C_t, 1 - L_t)$

- Example: Logarithmic utility function

$$U(C_t, 1 - L_t) = \gamma \ln C_t + (1 - \gamma) \ln(1 - L_t) \tag{21}$$

where $\gamma \in (0, 1)$ is the weight of consumption.

- In this particular case the household problem is given by:

$$\max_{\{C_t, L_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \left[\gamma \ln C_t + (1 - \gamma) \ln(1 - L_t)\right] \tag{22}$$

subject to the budget constraint and the initial and final conditions.

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- Lagrangian auxiliary function:

$$\mathcal{L} = \sum_{t=0}^{T} \left[\begin{array}{c} \beta^t \left[\gamma \ln C_t + (1-\gamma) \ln(1-L_t)\right] \\ -\lambda_t (C_t + B_t - W_t L_t - (1+R_{t-1}) B_{t-1}) \end{array} \right] \tag{23}$$

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \beta^t \frac{1}{C_t} - \lambda_t = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} : -\beta^t (1-\gamma) \frac{1}{1-L_t} + \lambda_t W_t = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} : -\lambda_t + \lambda_{t+1} (1+R_t) = 0 \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : C_t + B_t - W_t - (1+R_{t-1}) B_{t-1} = 0 \tag{27}$$

- Solving for the Lagrange multiplier in the first first order condition and substituting in the third first order condition, we have that:

$$\beta^t \frac{1}{C_t} = \beta^{t+1} \frac{1}{C_{t+1}}(1+R_t) \tag{28}$$

and operaing, the optimal consumption path is given by:

$$C_{t+1} = \beta(1+R_t)C_t \tag{29}$$

- Finally, by substuting the Lagrange multiplier in the second first order condition we obtain:

$$\frac{1-\gamma}{\gamma}\frac{C_t}{1-L_t} = W_t \tag{30}$$

that is, the equilibrium condition representing the labor supply. This condition indicates the number of hours the households will devote to working activities as a function of the cost of leisure (measured by the salary per unit of time).

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Household problem solution **Logarithmic utility funciton for consumption and leisure**	
Optimal consumption path	$C_{t+1} = \beta(1+R_t)C_t$
Financial assets dynamics	$B_t = (1+R_{t-1})B_{t-1} + W_t L_t - C_t$
Labor supply	$(1-\gamma)C_t = \gamma W_t(1-L_t)$