Lecture 6: The exchange rate overshooting

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Advanced Macroeconomics

6. The exchange rate overshooting

- Small open economy.
- Equilibrium relationship between domestic and foreign monetary markets: Uncovered Interest Parity.
- Equilibrium relationship between domestic and foreign goods and services markets: Purchasing Power Parity (only in the long-run).
- Puzzle:

$$\uparrow m \Longrightarrow \uparrow p \Longrightarrow \uparrow s$$
$$\uparrow m \Longrightarrow \downarrow i \Longrightarrow \downarrow s$$

Model economy:

$$m_t - p_t = \psi y_t - \theta i_t \tag{1}$$

$$y_t^d = \beta_0 + \beta_1 (s_t - p_t + p_t^*) - \beta_2 i_t \tag{2}$$

$$\Delta p_t = \mu (y_t^d - y_t^n) \tag{3}$$

$$\Delta s_t = i_t - i_t^* \tag{4}$$

where s is the log of the nominal exchange rate, p^*, is the log of the foreign price level, and i^* is the foreign nominal interest rate.

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Endogenous and exogenous variables.

1. Money supply (exogenous)
2. Domestic price level (endogenous)
3. Domestic output (endogenous)
4. Domestic nominal interest rate (endogenous)
5. Domestic aggregate demand (endogenous)
6. Nominal exchange rate (endogenous)
7. Foreign price level (exogenous)
8. Domestic potential output (exogenous)
9. Foreign nominal interest rate (exogenous)

Endogenous and exogenous variables.

- The model has 5 endogenous variables but only 4 equations.
- One missing equation.
- Or too many endogenous variables.
- We will eliminate one endogenous variable: Output.
- Assumption: Output is equal to potential output.

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Endogenous variables of reference.

1. Domestic price level
2. Nominal exchange rate

Model economy:

$$m_t - p_t = \psi y_t^n - \theta i_t \tag{5}$$

$$y_t^d = \beta_0 + \beta_1 (s_t - p_t + p_t^*) - \beta_2 i_t \tag{6}$$

$$\Delta p_t = \mu (y_t^d - y_t^n) \tag{7}$$

$$\Delta s_t = i_t - i_t^* \tag{8}$$

Difference equations.
Solving from the domestic nominal interest rate from equation (5):

$$i_t = -\frac{1}{\theta}(m_t - p_t - \psi y_t^n) \tag{9}$$

Substituting (9) in (6):

$$y_t^d = \beta_0 + \beta_1(s_t - p_t + p_t^*) + \frac{\beta_2}{\theta}(m_t - p_t - \psi y_t^n) \tag{10}$$

Difference equations.

$$\Delta p_t = \mu\beta_0 + \mu\beta_1 s_t + \mu\beta_1 p_t^* - \mu(\beta_1 + \frac{\beta_2}{\theta})p_t + \frac{\mu\beta_2}{\theta} m_t - \mu(\frac{\psi\beta_2}{\theta} + 1)y_t^n \tag{11}$$

$$\Delta s_t = -\frac{1}{\theta}(m_t - p_t - \psi y_t^n) - i_t^* \tag{12}$$

We can reduce the number of exogenous variables without affecting the model economy: For example, we can normalize the foreign price level to 1. Thus, $p_t^* = 0$.

Model in matrix notation.

$$
\begin{bmatrix} \Delta p_t \\ \Delta s_t \end{bmatrix} = \underbrace{\begin{bmatrix} -\mu(\beta_1 + \frac{\beta_2}{\theta}) & \mu\beta_1 \\ \frac{1}{\theta} & 0 \end{bmatrix}}_{A} \begin{bmatrix} p_t \\ s_t \end{bmatrix} +
$$

$$
\underbrace{\begin{bmatrix} \mu & \frac{\mu\beta_2}{\theta} & -\mu(\frac{\psi\beta_2}{\theta}+1) & 0 & \mu\beta_1 \\ 0 & -\frac{1}{\theta} & \frac{\psi}{\theta} & -1 & 0 \end{bmatrix}}_{B} \begin{bmatrix} \beta_0 \\ m_t \\ y_t^n \\ i_t^* \\ p_t^* \end{bmatrix} \tag{13}
$$

Structure of the Dornbusch's model	
Money demand	$m_t - p_t = \psi y_t - \theta i_t$
Aggregate demand	$y_t^d = \beta_0 + \beta_1 (s_t - p_t + p_t^*) - \beta_2 i_t$
Price adjustment	$\Delta p_t = \mu (y_t^d - y_t^n)$
Uncovered Interest Parity	$\Delta s_t = i_t - i_t^*$
Output	$y_t = y_t^n$
Inflation	$\Delta p_t = p_{t+1} - p_t$
Depreciation rate	$\Delta s_t = s_{t+1} - s_t$

Calibration:

Calibration of the parameters

Symbol	*Definition*	*Value*
ψ	Money demand elasticity	0.05
θ	Interest rate semi-elasticity	0.5
β_1	y_t^d elasticity to real exhange rate	20
β_2	y_t^d elasticity to interest rate	0.1
μ	Price adjustment velocity	0.01

Calibration:

Exogenous variables

Variable	*Definition*	*Value*
m_0	Money	100
β_0	Government spending	500
y_0^n	Potential output	2,000
p_0^*	Foreign price level	0
i_0^*	Foreign nominal interest rate	3

Steady state:

$$\begin{bmatrix} \overline{p}_t \\ \overline{s}_t \end{bmatrix} = -A^{-1} B \mathbf{z}_t$$

where:

$$\begin{aligned} A &= \begin{bmatrix} -\mu(\beta_1 + \frac{\beta_2}{\theta}) & \mu\beta_1 \\ \frac{1}{\theta} & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} \mu & \frac{\mu\beta_2}{\theta} & -\mu(\frac{\psi\beta_2}{\theta} + 1) & 0 & \mu\beta_1 \\ 0 & -\frac{1}{\theta} & \frac{\psi}{\theta} & -1 & 0 \end{bmatrix}, \\ \mathbf{z}_t &= \begin{bmatrix} \beta_0 \\ m_t \\ y_t^n \\ i_t^* \\ p_t^* \end{bmatrix} \end{aligned}$$

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Steady state:
Inverse of the matrix A:

$$A^{-1} = \begin{bmatrix} 0 & \theta \\ \frac{1}{\mu\beta_1} & \frac{\beta_1\theta+\beta_2}{\beta_1} \end{bmatrix}$$

Resulting in:

$$-A^{-1}B = \begin{bmatrix} 0 & 1 & -\psi & \theta & 0 \\ -\frac{1}{\beta_1} & 1 & \frac{(1-\beta_1\psi)}{\beta_1} & \frac{\beta_1\theta+\beta_2}{\beta_1} & -1 \end{bmatrix}$$

Operating:

$$\overline{p}_t = m_t - \psi y_t^n + \theta i_t^* \tag{14}$$

$$\overline{s}_t = m_t - \frac{\beta_0}{\beta_1} + \left[\frac{1-\psi\beta_1}{\beta_1}\right] y_t^n + \frac{\theta\beta_1 + \beta_2}{\beta_1} i_t^* - p_t^* \tag{15}$$

Using the calibrated values for the parameters and exogenous variables, we obtain:

$$\overline{p}_t = 100 - 0.05 \times 2000 + 0.5 \times 3 = 1.50$$

$$\overline{s}_t = 100 - \frac{500}{20} - \left[\frac{1 - 0.05 \times 20}{20}\right] \times 2000 + \frac{0.5 \times 20 + 0.1}{0.1} \times 3 - 0 = 76.52$$

Stability analysis.

$$Det \begin{bmatrix} -\mu(\beta_1 + \frac{\beta_3}{\theta}) - \lambda & \mu\beta_1 \\ \frac{1}{\theta} & 0 - \lambda \end{bmatrix} = 0 \tag{16}$$

$$\lambda^2 + \lambda \left[\beta_1\mu + \frac{\beta_3\mu}{\theta} \right] - \frac{\beta_1\mu}{\theta} = 0 \tag{17}$$

$$\frac{-(\beta_1\mu + \frac{\beta_3\mu}{\theta}) \pm \sqrt{\left[(\beta_1\mu + \frac{\beta_3\mu}{\theta}) \right]^2 + \frac{4\beta_1\mu}{\theta}}}{2} \tag{18}$$

Therefore, $\lambda_1 < 0, \lambda_2 > 0$.

$$\lambda_1 = \frac{-(0.2 + \frac{0.1 \times 0.01}{0.5}) - \sqrt{\left[(0.2 + \frac{0.1 \times 0.01}{0.5})\right]^2 + \frac{4 \times 20 \times 0.01}{0.5}}}{2} = -0.74$$

$$\lambda_2 = \frac{-(0.2 + \frac{0.1 \times 0.01}{0.5}) + \sqrt{\left[(0.2 + \frac{0.1 \times 0.01}{0.5})\right]^2 + \frac{4 \times 20 \times 0.01}{0.5}}}{2} = 0.54$$

Threfore $|\lambda_1 + 1| < 1, |\lambda_2 + 1| > 1$. Saddle point solution.

Saddle stable path

Let's start from the dynamic equation for the nominal exchange rate.

$$\Delta s_t = -\frac{1}{\theta}(m_t - p_t - \psi \bar{y}_t) - i_t^* \tag{19}$$

Define the equation for the stable path:

$$\Delta s_t = \lambda_1 (s_t - \bar{s}_t) \tag{20}$$

Equalizing both equations:

$$\lambda_1 (s_t - \bar{s}_t) = -\frac{1}{\theta}(m_t - p_t - \psi \bar{y}_t) - i_t^* \tag{21}$$

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Solving for the nominal exchange rate:

$$s_t = \frac{-(m_t - p_t - \psi \overline{y}_t)}{\theta \lambda_1} - \frac{i_t^*}{\lambda_1} + \overline{s}_t \tag{22}$$

For example, in the case of a change in money supply, the derivative of the nominal exchange rate with respect to this shock (this is the expectations adjustment):

$$\frac{ds_t}{dm_t} = \frac{-1}{\theta \lambda_1} + \frac{d\overline{s}_t}{dm_t} = 1 - \frac{1}{\theta \lambda_1} > 1 \tag{23}$$