

Lecture 4: A dynamic IS-LM model

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4. A dynamic IS-LM model

- Model economy:

$$m_t - p_t = \psi y_t - \theta i_t \quad (1)$$

$$y_t^d = \beta_0 - \beta_1 (i_t - \Delta p_t^e) \quad (2)$$

$$\Delta p_t = \mu (y_t - y_t^n) \quad (3)$$

$$\Delta y_t = v (y_t^d - y_t) \quad (4)$$

where m is the logarithm of money, p is the logarithm of prices, y^d , the logarithm of aggregate demand, y logarithm of output, y_t^n the logarithm of potential output, i is the nominal interest rate. All the parameters (greek letters) are defined as positive. Δ represents time derivative.

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Steps to follow:

- ① Endogenous versus exogenous variables.
- ② Selection of the endogenous variables of reference.
- ③ Setting-up the difference equations.
- ④ Matrix notation of the system.
- ⑤ Calibration of the model.
- ⑥ Steady State.
- ⑦ Stability analysis.
- ⑧ Numerical computation of the model.
- ⑨ Shocks analysis.
- ⑩ Sensitivity analysis.

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Step 1: Endogenous and exogenous variables.

- ① Money
- ② Price level
- ③ Potential output
- ④ Nominal interest rate
- ⑤ Aggregate demand
- ⑥ Output

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Step 2: Selection of the endogenous variables of reference.

- ① Price level
- ② Output

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Step 3: Difference equations.

Solving for the nominal interest rate in equation (1):

$$i_t = -\frac{1}{\theta}(m_t - p_t - \psi y_t) \quad (5)$$

Substituting (5) in (2):

$$y_t^d = \beta_0 - \frac{\beta_1 \psi}{\theta} y_t + \frac{\beta_1}{\theta}(m_t - p_t) + \beta_1 \Delta p_t^e \quad (6)$$

Perfect foresight assumption ($\Delta p_t = \Delta p_t^e$):

$$y_t^d = \beta_0 - \frac{\beta_1 \psi}{\theta} y_t + \frac{\beta_1}{\theta}(m_t - p_t) + \beta_1 \Delta p_t \quad (7)$$

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Step 3: Difference equations.

Substituting aggregate demand in the dynamic equation for output:

$$\dot{y}_t = v \left[\beta_0 - \left(\frac{\beta_1 \psi}{\theta} + 1 \right) y_t + \frac{\beta_1}{\theta} (m_t - p_t) + \beta_1 \Delta p_t \right] \quad (8)$$

$$\dot{y}_t = v \left[\beta_0 - \left(\frac{\beta_1 \psi}{\theta} + 1 \right) y_t + \frac{\beta_1}{\theta} (m_t - p_t) + \beta_1 \mu (y_t - y_t^n) \right] \quad (9)$$

$$\dot{y}_t = v \left[\beta_0 + \left(\beta_1 \mu - \frac{\beta_1 \psi}{\theta} - 1 \right) y_t + \frac{\beta_1}{\theta} (m_t - p_t) - \beta_1 \mu y_t^n \right] \quad (10)$$

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Step 4: Model in matrix notation.

$$\dot{p}_t = \mu(y_t - y_t^n) \quad (11)$$

$$\dot{y}_t = v \left[\beta_0 + (\beta_1 \mu - \frac{\beta_1 \psi}{\theta} - 1)y_t + \frac{\beta_1}{\theta}(m_t - p_t) - \beta_1 \mu y_t^n \right] \quad (12)$$

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Step 4: Model in matrix notation.

$$\begin{bmatrix} \Delta p_t \\ \Delta y_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \mu \\ \frac{-v\beta_1}{\theta} & v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) \end{bmatrix}}_A \begin{bmatrix} p_t \\ y_t \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & -\mu \\ v & \frac{v\beta_1}{\theta} & -v\beta_1\mu \end{bmatrix}}_B \begin{bmatrix} \beta_0 \\ m_t \\ y_t^n \end{bmatrix} \quad (13)$$

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Structure of the dynamic IS-LM model

Monetary market	$m_t - p_t = \psi y_t - \theta i_t$
Goods and services market	$y_t^d = \beta_0 - \beta_1(i_t - \Delta p_t^e)$
Price level adjustment	$\Delta p_t = \mu(y_t - y_t^n)$
Output adjustment	$\Delta y_t = v(y_t^d - y_t)$
Inflation	$\Delta p_t = p_{t+1} - p_t$
Output growth	$\Delta y_t = y_{t+1} - y_t$

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Step 5: Calibration

Calibration of the parameters

Symbol	Definition	Value
ψ	Elasticity of $m_t - p_t$ to output	0.05
θ	Interest rate semi-elasticity	0.5
β_1	Elasticity of y_t^d to interest rate	50
μ	Price level adjustment rate	0.01
v	Output adjustment rate	0.2

4. El modelo IS-LM dinámico

Step 5: Calibration.

Values for the exogenous variables

Variable	Definition	Value
m_0	Money supply	100
β_0	Autonomous component of y_t^d	2,100
y_0^n	Potential output	2,000

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Step 6: Steady State.

Steady state is computed as:

$$\begin{bmatrix} \bar{p}_t \\ \bar{y}_t \end{bmatrix} = -A^{-1}Bz_t \quad (14)$$

Using the above expression we obtain:

$$\bar{p}_t = \frac{\theta\beta_0}{\beta_1} + m_t - \left(\psi + \frac{\theta}{\beta_1}\right)y_t^n \quad (15)$$

$$\bar{y}_t = y_t^n \quad (16)$$

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Step 6: Steady State.

Given the calibration of the model we have:

$$\begin{bmatrix} \bar{p}_t \\ \bar{y}_t \end{bmatrix} = - \begin{bmatrix} 0 & 0.01 \\ -20 & -1.1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & -0.01 \\ 0.2 & 20 & -0.1 \end{bmatrix} \begin{bmatrix} 2,100 \\ 100 \\ 2,000 \end{bmatrix} =$$
$$= \begin{bmatrix} 0.5/50 & 1 & -0.05 - 0.5/50 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2,100 \\ 100 \\ 2,000 \end{bmatrix} = \begin{bmatrix} 1 \\ 2,000 \end{bmatrix}$$

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Step 6: Steady State.

- Using the definition of the nominal interest rate, we have:

$$\bar{i}_0 = -\frac{1}{0.5}(100 - 1 - 0.05 \times 2,000) = 2$$

- The corresponding value for the steady state aggregate demand is:

$$\bar{y}_0^d = \beta_0 - \beta_1(i_0 - \Delta p_0) = 2,100 - 50 \times 2 = 2,000$$

given that in steady state $\Delta p_0 = 0$.

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Step 7: Stability analysis.

$$\text{Det} \begin{bmatrix} 0 - \lambda & \mu \\ \frac{-v\beta_1}{\theta} & v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) - \lambda \end{bmatrix} = 0 \quad (17)$$

$$\lambda^2 - \lambda \left[v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) \right] + \frac{v\beta_1\mu}{\theta} = 0 \quad (18)$$

$$\frac{v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) \pm \sqrt{\left[v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) \right]^2 - \frac{4v\beta_1\mu}{\theta}}}{2} \quad (19)$$

If $\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1 > 0 \implies \lambda_1 > 0, \lambda_2 > 0$.

If $\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1 < 0 \implies \lambda_1 < 0, \lambda_2 < 0$.

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Step 7: Stability analysis.

- Notice that if the term $\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1$ is positive, then the two roots are also positive ($\lambda_1 > 0, \lambda_2 > 0$). If this is the case, then all trajectories are explosive, as the modulus of the root plus one will be larger than one, $|\lambda_1 + 1| > 1$ and $|\lambda_2 + 1| > 1$. Therefore, that term cannot be negative.

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Step 7: Stability analysis.

- Using the calibrated values for the parameters we have that:

$$\beta_1 \mu - \frac{\beta_1 \psi}{\theta} - 1 = 50 \times 0.01 - \frac{50 \times 0.05}{0.5} - 1 = -5.5 < 0$$

and therefore, that term is negative.

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Step 7: Stability analysis.

- Next, we check the sign of the term inside the square root:

$$\left[v\left(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1\right) \right]^2 - \frac{4v\beta_1\mu}{\theta} \leq 0$$

- Using the calibrated values for the parameters, we have that:

$$-1.1^2 - 0.8 > 0$$

and thus, the two eigenvalues are real numbers (for this particular calibration).

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Step 7: Stability analysis.

- Finally, we compute the value for the two eigenvalues:

$$\lambda_1 = \frac{v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) + \sqrt{\left[v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1)\right]^2 - \frac{4v\beta_1\mu}{\theta}}}{2} = -0.23$$

$$\lambda_2 = \frac{v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1) - \sqrt{\left[v(\beta_1\mu - \frac{\beta_1\psi}{\theta} - 1)\right]^2 - \frac{4v\beta_1\mu}{\theta}}}{2} = -0.87$$

resulting in two negative values, where the modulus plus one are 0.77 and 0.13, both lower than one, resulting in global stability.