

Lecture 16: Revisiting the Solow model

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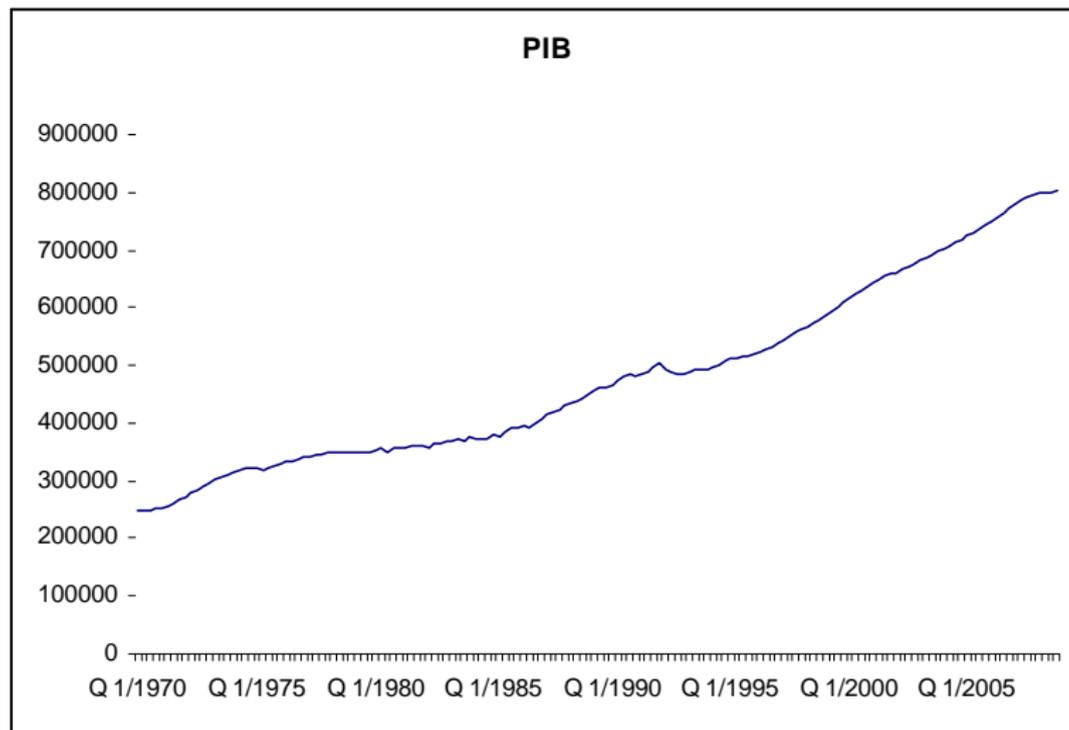
Universidad de Málaga

Advanced Macroeconomics

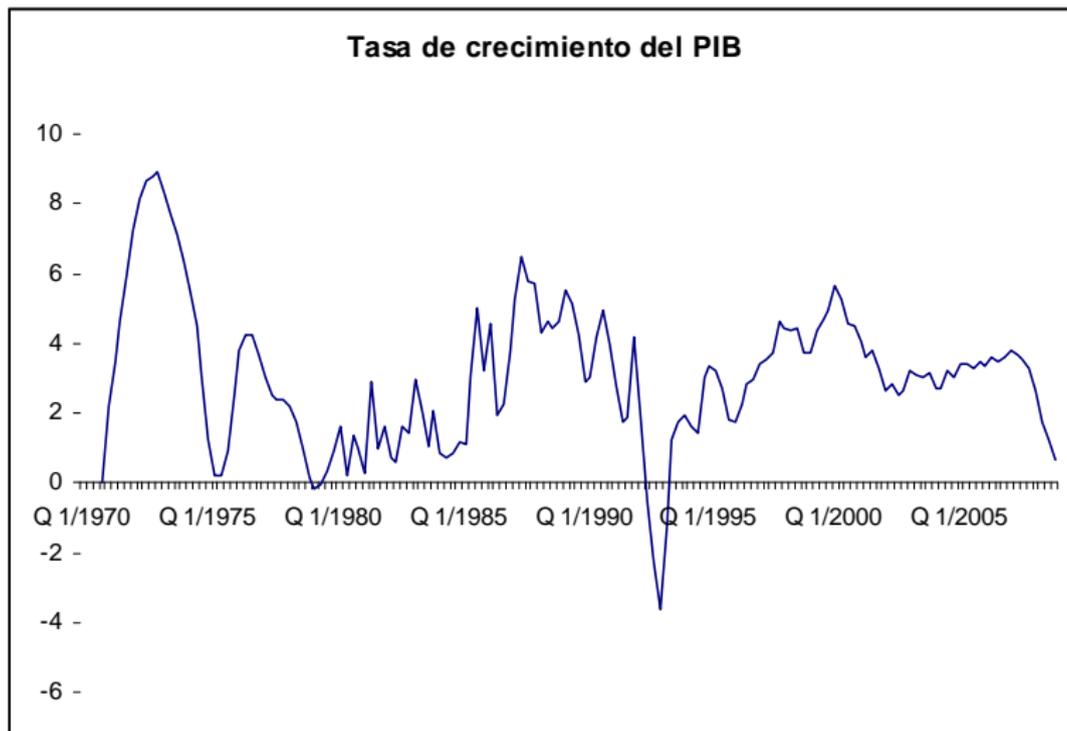
16. Revisiting the exogenous growth model

- Important questions:
 - Why economies growth?
 - Why some economies have larger income level than others?
 - Why some economies growth at high rates and other economies at low rates?
 - What is a economic miracle?
 - Is possible that today rich countries will be poor countries in the future? And the opposite?
 - Is possible that some poor countries will remain poor forever?

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- Richer countries across time:
- Around 1600: China.
- Around 1700: Italy.
- XIX Century: Europe.
- From 1900: United States

GDP (billions of dollars PPP) in 2010 (Source: IMF)

1	USA	14,526,000
2	China	10,119,000
3	Japan	4,323,000
4	India	4,057,000
5	Germany	2,944,000
6	Russia	2,230,000
7	UK	2,181,000
8	Brazil	2,178,000
9	France	2,135,000
10	Italy	1,779,000
11	Mexico	1,564,000
12	South Korea	1,466,000
13	Spain	1,372,000

GDP per capita (PPP dollars) in 2010 (Source: IMF)

1	Luxemburg	81,466
2	Singapore	56,694
3	Norway	51,959
4	USA	46,860
5	Switzerland	41,950
6	Holland	40,973
7	Australia	39,764
8	Austria	39,761
9	Ireland	39,492
10	Taiwan	39,245
11	Canada	39,171
12	Sweden	38,204

Relative GDP per capita

1	USA	100.0	14	Holland	74.0
2	Luxembourg	93.6	16	Austria	72.0
3	Hong Kong	91.8	16	UK	70.9
4	Canada	91.2	17	Italy	70.9
5	Switzerland	88.5	18	Singapore	70.5
6	Norway	86.5	19	Iceland	70.3
7	Japan	84.2	20	Finland	66.9
8	Germany	82.0	21	New Zealand	63.3
9	Australia	80.6	22	Israel	54.9
10	Denmark	78.5	23	Spain	54.6
11	Sweden	77.9	24	Ireland	53.7
12	France	77.6	25	Chyprus	51.3
13	Belgium	75.1			

Source: Penn World Tables

Relative GDP per capita

1	USA	100.0
37	Brazil	21.6
43	Colombia	18.8
55	Morocco	12.1
57	Peru	11.7
67	India	7.1
72	Nigeria	5.4
80	Mozambique	4.0
85	Uganda	3.0
90	Chad	2.3

Source: Penn World Tables

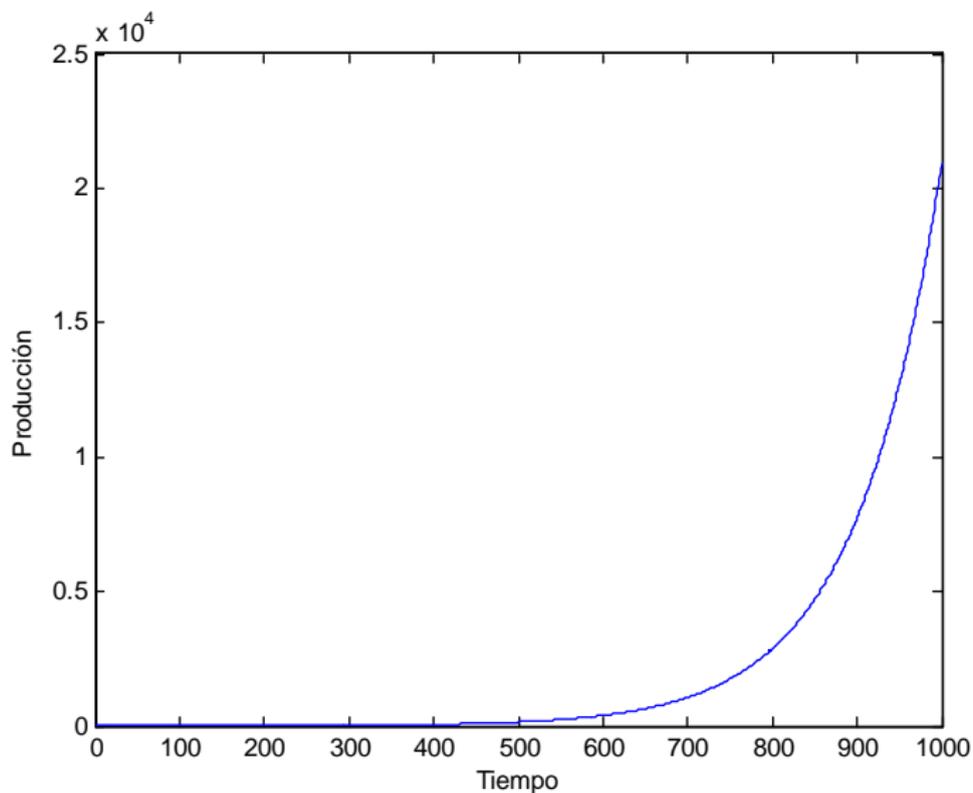
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- Economic growth is an accumulative process.
- Assuming an average annual growth rate of 1% from the year 0, the current GDP would be of **485 millions**:

$$(1 + 0,01)^{2010} = 485.245.261 \quad (1)$$

- If a Roman could buy one computer in the year 0, today we can buy 485 millions of computers

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- The rise in GDP during a particular period can be calculated as:
 $(1 + g)^t$
- If the growth rate is g the level of GDP doubles approximately every $70/g$ years:
- Spain. Average growth rate: 2%. Spain doubles GDP every 35 years.
- South Korea. Average growth rate: 6%. South Korea doubles GDP every 11.6 years.
- China. Average growth rate: 16%. China doubles GDP every 4.6 years.

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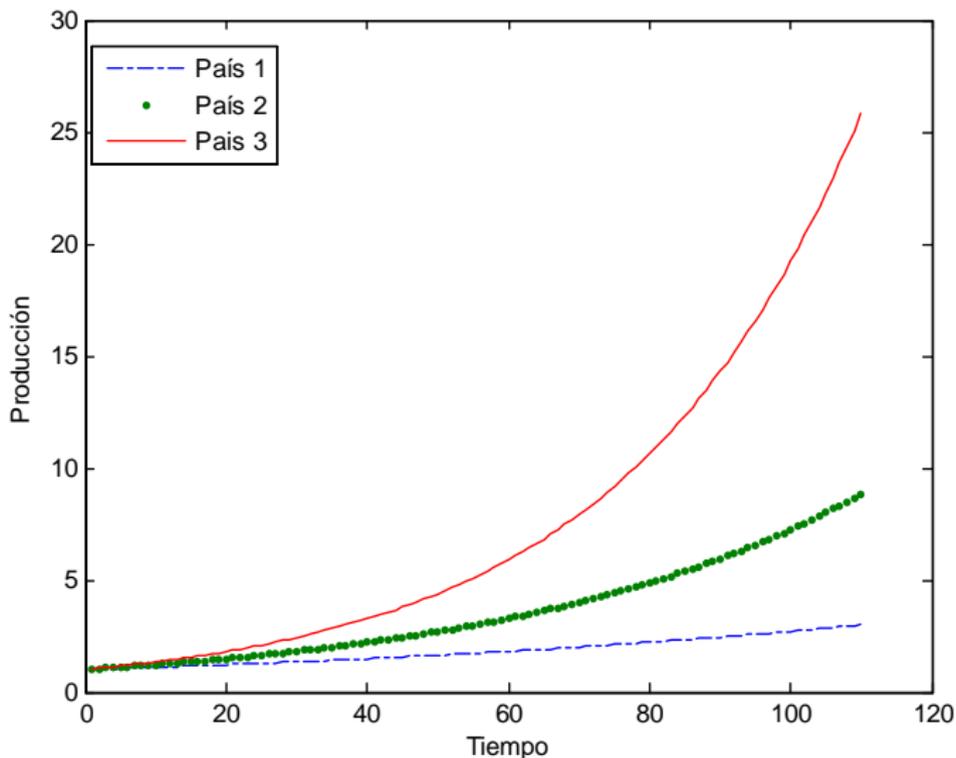
- Consider three economies with the same level of GDP in the initial period, but with different growth rates. After 110 years we obtain:

$$(1 + 0,01)^{110} = 2,98 \quad (\text{Country 1})$$

$$(1 + 0,02)^{110} = 8,83 \quad (\text{Country 2})$$

$$(1 + 0,03)^{110} = 25,82 \quad (\text{Country 3})$$

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- Aggregate production functions:

$$Y_t = A_t F(K_t, L_t) \quad (2)$$

- Assumption: Constant Return to Scale.
- Capital accumulation equation:

$$\dot{K}_t = I_t - \delta K_t = sY_t - \delta K_t \quad (3)$$

- Assumption: Exogenous saving rate.

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- Population is increasing over time:

$$L_t = L_0(1 + n)^t; \quad n > 0 \quad (4)$$

- Assumption: Population growth is exogenous.

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In per capita terms:

$$\frac{Y_t}{L_t} = \frac{A_t K_t^\alpha L_t^{1-\alpha}}{L_t} = \frac{A_t K_t^\alpha L_t^1 L_t^{-\alpha}}{L_t} = A_t K_t^\alpha L_t^{-\alpha} = \frac{A_t K_t^\alpha}{L_t^\alpha} = A_t \left(\frac{K_t}{L_t} \right)^\alpha \quad (5)$$

$$y_t = A_t k_t^\alpha \quad (6)$$

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- Per capita production function:

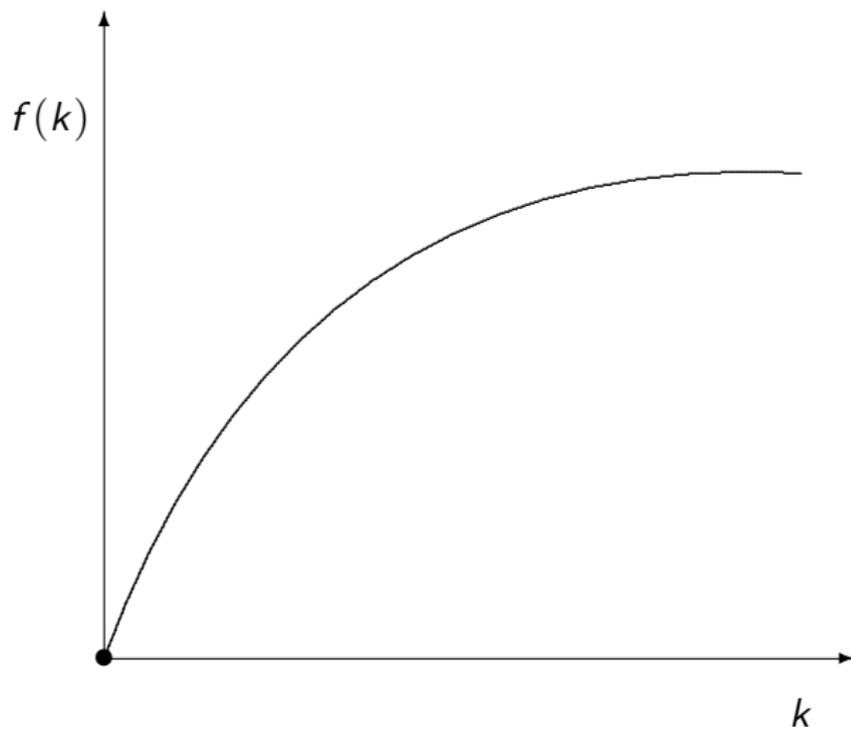
$$y_t = f(k_t) \quad (7)$$

$$y_t = \frac{Y_t}{L_t} \quad (8)$$

- Capital stock per capita:

$$k_t = \frac{K_t}{L_t} \quad (9)$$

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Capital accumulation is given by:

$$\Delta K_t = Y_t - C_t - \delta K_t \quad (10)$$

where $\Delta K_t = K_{t+1} - K_t$. Operating:

$$\frac{C_t + K_{t+1} - (1 - \delta)K_t}{L_t} = \frac{Y_t}{L_t} \quad (11)$$

$$\frac{C_t}{L_t} + \frac{K_{t+1}L_{t+1}}{L_tL_{t+1}} - \frac{(1 - \delta)K_t}{L_t} = \frac{Y_t}{L_t} \quad (12)$$

$$\frac{C_t}{L_t} + \frac{K_{t+1}}{L_{t+1}} \frac{L_{t+1}}{L_t} - \frac{(1 - \delta)K_t}{L_t} = \frac{Y_t}{L_t} \quad (13)$$

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On the other hand:

$$\frac{L_{t+1}}{L_t} = (1 + n) \quad (14)$$

resulting

$$c_t + k_{t+1}(1 + n) - (1 - \delta)k_t = y_t \quad (15)$$

Solving for capital stock per capita in $t + 1$:

$$k_{t+1} = \frac{(1 - \delta)k_t + y_t - c_t}{(1 + n)} \quad (16)$$

Using the definition of investment ($y_t - c_t = i_t = s_t y_t$):

$$k_{t+1} = \frac{(1 - \delta)k_t + s_t y_t}{(1 + n)} \quad (17)$$

Operating:

$$k_{t+1} - k_t + k_t = \frac{(1 - \delta)k_t + s_t y_t}{(1 + n)} \quad (18)$$

or:

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Solow-Swan model	
Production function	$Y_t = A_t F(K_t, L_t)$
Capital accumulation	$K_{t+1} = (1 - \delta) K_t + I_t$
Initial capital stock	$K_0 > 0$
Feasibility condition	$Y_t = C_t + I_t$
Investment	$I_t = s_t Y_t$
Population dynamics	$L_t = L_0(1 + n)^t$

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- Steady State:

$$s_t \bar{y}_t = (\delta + n) \bar{k}_t \quad (21)$$

Assuming that technology is Cobb-Douglas:

$$s_t A_t \bar{k}_t^\alpha = (\delta + n) \bar{k}_t \quad (22)$$

$$\bar{k}_t^{\alpha-1} = \frac{\delta + n}{s_t A_t} \quad (23)$$

$$\bar{k}_t = \left(\frac{\delta + n}{s_t A_t} \right)^{\frac{1}{\alpha-1}} \quad (24)$$

$$\bar{y}_t = A_t \bar{k}_t^\alpha = A_t \left(\frac{\delta + n}{s_t A_t} \right)^{\frac{\alpha}{\alpha-1}} \quad (25)$$

$$\bar{c}_t = (1 - s_t) \bar{y}_t = (1 - s_t) A_t \left(\frac{\delta + n}{s_t A_t} \right)^{\frac{\alpha}{\alpha-1}} \quad (26)$$