Lecture 10: The government

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Advanced Macroeconomics

10. The government

- We introduce a new agent into the economy: The government.
- This agent takes decisions about fiscal policy: Taxes and public spending.
- Models consider the government as an exogenous agent to the economy.

10. The government: Non-distortionary taxation

- Household maximization problem:

$$\max_{\{C_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t U(C_t) \tag{1}$$

- Budget constraint:

$$C_t + B_t = (1 - \tau_t^w) W_t + (1 + R_{t-1}) B_{t-1} + G_t \tag{2}$$

where τ_t^w is the labor income tax rate. We assume that the government returns to households fiscal income as a lump-sum transfer, where G_t is the lump-sum transfer (public spending), which are assumed to be equal to fiscal revenues, $G_t = \tau_t^w W_t$. All these are exogenous variables.

10. The government: Non-distortionary taxation

- Alternatively, we can assume that lump-sum transfers are zero $(G_t = 0)$, that is, the government burns the money from fiscal revenues. In this case, the budget constraint is given by:

$$C_t + B_t = (1 - \tau_t^w) W_t + (1 + R_{t-1}) B_{t-1} \tag{3}$$

10. The government: Non-distortionary taxation

Non-distortionary taxation	
Utility function	$U = U(C_t)$
Budget constraint	$C_t + B_t = (1 + R_{t-1})B_{t-1}$
	$+(1 - \tau_t^w)W_t + G_t$
Initial stock of saving	$B_{-1} = 0$
Final stocl of saving	$B_T = 0$
Government budget constraint	$G_t = \tau_t^w W_t$

10. The government: Non-distortionary taxation

- Logarithmic utility function:

$$U(C_t) = \ln C_t \tag{4}$$

- Auxiliary Lagrangian function:

$$\mathcal{L} = \sum_{t=0}^{T} \left[\beta^t \ln C_t - \lambda_t (C_t + B_t - (1 - \tau_t^w) W_t - (1 + R_t) B_{t-1}) - G_t \right] \tag{5}$$

10. The government: Non-distortionary taxation

First order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &: \quad \frac{\beta^t}{C_t} - \lambda_t = 0 & (6)\\
\frac{\partial \mathcal{L}}{\partial B_t} &: \quad -\lambda_t + \lambda_{t+1}(1+R_t) = 0 & (7)\\
\frac{\partial \mathcal{L}}{\partial \lambda_t} &: \quad C_t + B_t - \beta(1-\tau_t^w)W_t - (1+R_{t-1})B_{t-1} - G_t = 0 & (8)
\end{aligned}
$$

- By substituting the Lagrange multiplier from the first first order condition into the second first order condition, we get

$$\beta^t \frac{1}{C_t} = \beta^{t+1} \frac{1}{C_{t+1}} (1 + R_t) \tag{9}$$

and operating results the following optimal consumption path (no sign of the tax):

$$C_{t+1} = \beta(1 + R_t) C_t \tag{10}$$

Solution to the household problem with taxes Non-distortionary taxes. Logarithmic utility function	
Optimal consumption tax	$C_{t+1} = \beta(1 + R_t)C_t$
Financial assets dynamics	$B_t = (1 + R_{t-1})B_{t-1}$
	$+(1 - \tau_t^w)W_t + G_t - C_t$

10. The government: Distortionary taxation

- We consider the existence of three taxes: Consumption tax, labor income tax, and capital tax (saving tax).
- We consider leisure in a logarithmic utility function. Household maximization problem is defined as:

$$\max_{\{C_t, L_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t (\gamma \ln C_t + (1-\gamma) \ln(1-L_t)) \tag{11}$$

- Budget constraint will be:

$$(1+\tau_t^c)C_t + B_t = (1-\tau_t^w)W_t L_t + \left[1 + (1-\tau_t^r)R_{t-1}\right] B_{t-1} + G_t \tag{12}$$

where, τ_t^c is the consumption tax rate, τ_t^w is the labor income tax rate, and τ_t^r is the capital income tax rate.

Household problem: Distortionary taxation	
Utility function	$U = U(C_t, O_t)$
Budget constraint	$(1+\tau_t^c)C_t + B_t = (1-\tau_t^w)W_t L_t$
	$+\left[1+(1-\tau_t^r)R_{t-1}\right]B_{t-1} + G_t$
Initial stock of assets	$B_{t-1} = 0$
Final stock of assets	$B_T = 0$
Government budget constraint	$G_t = \tau_t^c C_t + \tau_t^w W_t L_t + \tau_t^r R_{t-1} B_{t-1}$

- Auxiliary Lagrange function:

$$\begin{aligned}\mathcal{L} &= \beta^t \left[\gamma \ln C_t + (1-\gamma)\ln(1-L_t)\right] \\ &\quad -\lambda_t \left[\begin{array}{c}(1+\tau_t^c)C_t + B_t - (1-\tau_t^w)W_t L_t \\ -\left[1+(1-\tau_t^r)R_{t-1}\right]B_{t-1} - G_t\end{array}\right]\end{aligned} \tag{13}$$

- First order conditions:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \frac{\gamma\beta^t}{C_t} - \lambda_t(1+\tau_t^c) = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} : -\lambda_t + \lambda_{t+1}(1-\tau_t^r)R_t = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} : -\frac{\beta^t(1-\gamma)}{1-L_t} + \lambda_t(1-\tau_t^w)W_t = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : (1+\tau_t^c)C_t + B_t - (1-\tau_t^w)W_tL_t - \left[1+(1-\tau_t^r)R_{t-1}\right]B_{t-1} - G_t = 0 \tag{17}$$

10. The government: Distortionary taxation

- By substituting the Lagrange multiplier from the first first order condition (14) into the second first order condition (15) we obtain:

$$\beta^t \frac{1}{(1+\tau_t^c)C_t} = \beta^{t+1} \frac{[1+(1-\tau_t^r)R_t]}{(1+\tau_{t+1}^c)C_{t+1}} \tag{18}$$

and operating, the optimal consumption path is given by:

$$(1+\tau_{t+1}^c)C_{t+1} = \beta\,[1+(1-\tau_t^r)R_t]\,(1+\tau_t^c)C_t \tag{19}$$

- Optimal consumption path is affected by the consumption tax and the capital income tax. However, if the consumption tax rate is constant over time (that is $\tau_{t+1}^c = \tau_t^c$), then the optimal consumption path would be:

$$C_{t+1} = \beta\,[1+(1-\tau_t^r)R_t]\,C_t \tag{20}$$

10. The government: Distortionary taxation

- Labor supply function (equilibrium condition for employment) is derived by:

$$\frac{1-\gamma}{\gamma}(1+\tau_t^c)C_t = (1-\tau_t^w)W_t(1-L_t) \tag{21}$$

10. The government: Distortionary taxation

- Labor supply function (equilibrium condition for employment) is derived by:

$$\frac{1-\gamma}{\gamma}(1+\tau_t^c)C_t = (1-\tau_t^w)W_t(1-L_t) \tag{22}$$

10. The government: Distortionary taxation

Solution fo the household problem with taxes **Logarithmic utility function**	
Optimal consumption tax	$(1+\tau^c_{t+1})C_{t+1} =$ $\beta\left[1+(1-\tau^r_t)R_t\right](1+\tau^c_t)C_t$
Financial assets dynamics	$B_t = \left[1+(1-\tau^r_t)R_{t-1}\right]B_{t-1}$ $+(1-\tau^w_t)W_t + G_t - (1+\tau^c_t)C_t$
Labor supply	$(1-\gamma)(1+\tau^c_t)C_t = \gamma(1-\tau^w_t)W_t(1-L_t)$