Sharing Economy in Macroeconomics: Collaborative Consumption and Durable Goods*

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Abstract

This paper studies the implications of the sharing economy on the rest of the economy over the business cycle. We propose a dynamic stochastic general equilibrium theoretical framework within which to study the collaborative economy sector, together with both the market production and household production sectors. The model considers that production within the collaborative economy falls between market production and domestic production, and combines some features of both environments, but differs in others. We find that a positive neutral technological shock to market production has a positive impact on the accumulation of business capital but reduces the stock of durables, reducing both home production and sharing economy production. Similarly, a positive productivity shock to the sharing economy sector increases investment in durables and sharing production, reducing business capital investment, market output, and market hours. Finally, an investment-specific technological shock to durables has a positive effect on household capital but a negative one on business capital. Interestingly, investment-specific technological shocks do not reduces effective consumption when a sharing economy and a home production sector are considered.

Keywords: Collaborative consumption; Sharing economy; Household production; Durable goods.

JEL Classification: D13; D16; E22.

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1 Introduction

The so-called collaborative or sharing economy has undergone significant growth in recent years, particularly in tourism and transportation activities. This expansion is largely attributable to technological advancements, specifically the development of Information and Communication Technologies (ICTs), which have eliminated traditional market barriers and information constraints (Fang et al., 2019), providig households with a marketplace to rent goods and services produces by using household capital. As a result, households have been able to reallocate resources that were previously reserved for the production at home of non-tradable goods and services towards the creation of tradeable goods and services that can be marketed and provided to other consumers. Digital intermediation and the emergence of platforms have increased the efficiency of matching supply and demand, as well as in the use of certain assets owned by households (Coyle, 2018). These assets take the form of durable goods and provide services that could be consumed by the members of the household or purchased by other people outside the household.

The importance of the sharing economy has been recognized by various authors and institutions, as well as its implications for GDP measurement (Ahmad and Schreyer, 2016, Office for National Statistics, 2017, International Monetary Fund, 2018, Ahmad and Ribarsky, 2018, Coyle, 2018, Barefoot et al. 2018, among others). According to these sources, sharing economy transactions have become increasingly relevant and significant in most economies around the world. Nevertheless, despite the growing recognition of its importance, to the best of our knowledge, no formal theoretical model has been proposed in the macroeconomic literature to study the implications of the collaborative economy on the rest of the economy. The sharing economy represents a significant new phenomenon in the economy, reflecting changes in the way goods and services are produced and consumed, as well as in the way assets are owned and valued. Therefore, there is a need for a theoretical framework that can capture the potential economic effects of the sharing economy and guide policy decisions in this area.

This paper presents a novel contribution by proposing a Dynamic Stochastic General Equilibrium (DSGE) model that characterizes the collaborative sector alongside the standard market and home production sectors. The model aims to address a significant gap in the literature by offering a comprehensive framework for analyzing the production factors and types of goods and services that are distinct to each sector and their interactions over the business cycle. Specifically, the proposed model acknowledges that collaborative production lies somewhere between market and domestic production, sharing some features of both environments while differing in others. We define the sharing or collaborative economy as consumer-to-consumer (C2C) transactions of goods and services that create an open marketplace for temporary access to those goods or services, without involving a change of ownership (Botsman and Rogers, 2010a, b; and Belk, 2014). Beladi et al. (2023) develop a model for a sharing economy with rural-urban migration to study the network effect on income distribution and social welfare and find that rural-urban migration is mitigated by realocating capital to the rural sector. This paper develops a DSGE model with a sharing economy to study their implications over the business cycle.

Our theoretical model extends a standard DSGE model with home production by incorporating the sharing sector. Home production is an important component of our framework, not only due to its relative size in terms of resource allocation and output, but also because household capital can be used for production in the sharing economy. We consider household spending on durable goods as physical capital, including residential structures (housing) and durables such as transport equipment. The size of household capital is quantitatively substantial and comparable to the stock of business capital. According to Greenwood and Hercowitz (1991), the average ratio between the two capital stocks in the U.S. between 1954 and 1988 was 1.13. The model assumes that goods and services produced in the market, at home, and in the sharing economy are not perfect substitutes, but there is a certain degree of substitution between them. The differences among these sectors are primarily based on the goods and services they produce, which are not homogeneous, and the inputs used in each corresponding production process.

The model is calibrated to conduct simulations. First, we calculate the steady-state of our model economy using a benchmark calibration to obtain quantitative results regarding the relative importance of the sharing economy sector compared to the other two sectors. Our analysis then identifies the effects of three neutral technological shocks specific to each sector and two investment-specific technological change (ISTC) shocks that affect the two types of capital. Specifically, the model examines how these shocks impact the production of goods and services in the market, at home, and in the sharing economy. By studying these effects, our aim is to gain a better understanding of the dynamic interactions among the different sectors and their impact on the economy as a whole.

The findings suggest that a positive productivity shock in the market sector provokes similar results to the standard home production model (see, for instance, Benhabid, Rogerson and Wright, 1991; Greenwood and Hercowitz, 1991; and McGrattan, Rogerson and Wright, 1997), by increasing market production and investment in business capital but reducing investment in household capital. This investment reallocation effect leads to a reduction in both home production and sharing output. Hence, incorporating a sharing economy into the model does not change the properties of the home production model in response to an aggregate productivity shock to the market sector. In contrast, a positive neutral technological shock to the sharing economy sector boosts sharing output and consumption, with a negligible effect on market output and home production. This shock also increases investment in household capital and reduces investment in business capital but of a small magnitude. In the case of a a positive productivity shock to home production, we find that both home production and sharing output increase in response to the shock. Hours in both the home production sector and the sharing economy sector increases, increasing also investment in durables, resulting in a negative impact on investment, hours, and output in the market sector. Lastly, ISTC shocks on durables have a negative impact on market output, triggering an increase in production in both the home and sharing economy sectors, and leading to a decrease in business capital investment. Interestingly, investment-specific technological shocks do not reduces effective consumption when a sharing economy and a home production sector are considered.

The remainder of this paper is organized as follows. Section 2 presents a DSGE model that incorporates the sharing sector. Section 3 presents the calibration of the model. Section 4 analyzes the effects of productivity shocks on the different sectors. Section 5 examines the impact of investment-specific technological shocks on the economy, specifically in the sharing sector. Finally, Section 6 concludes with some general conclusions.

2 The model

2.1 Model economy setup

Standard DSGE models divide households' discretionary available time in two parts: market work and leisure. However, households devote some inputs to activities at home, such as time devoted to home activities like cooking, child rearing, house cleaning or gardening which cannot be considered as leisure. Additionally, households can invest and accumulate capital in the form of durable goods, apart from business capital, to be used in the home production sector. All these activities represent home production of goods and services produced and consumed by households. Becker's (1965) seminal paper on the allocation of time showed that there was room for economic theory to go beyond the traditional work-leisure dichotomy. Following from this work, Gronau (1977) modifies Becker's framework to analyze home production and considers a trichotomy of time use: work in the market, work at home and leisure. Some authors have extended the basic model proposed by Gronau (1977) to a macroeconomic context, studying the implications of household productive activities in a dynamic general equilibrium model. Examples of

DSGE models with home production are Benhabid, Rogerson and Wright (1991), Greenwood and Hercowitz (1991) and McGrattan, Rogerson and Wright (1997). These models also consider the total capital stock composed of capital used in market production and in home production. The introduction of the household sector increases the explanatory power of the standard DSGE model.[1] One of the main implications is that households can increase the number of hours devoted to market work by decreasing time working at home, while keeping leisure time constant.[2] However, the model fails in predicting the procyclical behavior of durable goods when a Cobb-Douglas technology is used for home production.

Here, we extend previous works developing a model economy with three productive sectors with differentiating technologies: the standard productive market, the home production sector and the sharing economy sector. First, the model includes a standard market sector, which employs both labor and physical capital in the form of market structures, machinery and equipment for the production of market goods and services. Secondly, the domestic sector, in which a time other than market labor is used, but which we can not consider as leisure, and domestic capital, which is different from the productive capital used in the market sector. This domestic capital is composed of domestic structures (houses), and durable goods that constitute equipment (transport equipment in the case of vehicles and other equipment used for the production of domestic goods and services, such as refrigerators, washing machines, ovens and microwaves, among others). And thirdly, the collaborative sector, which produces and markets goods and services, but which uses household capital assets and additional labor time that is different from market labor and home production labor.

This is an integrated approach to modelling the collaborative economy as another productive sector of the economy; a sector characterized by combining some characteristics of both market and domestic productive sectors which render this distinction reasonable and consistent. The existing literature clearly distinguishes between productive physical capital (structures and equipment used by firms to produce goods and services traded in

[1] In particular, real business cycle models with explicit household production sectors perform better than the standard real business cycle model. However, as pointed out by McGrattan et al. (1997), the extent of this improvement depends critically on several parameters, including the elasticities of substitution between household and market variables in utility and production functions as well as the stochastic properties of the household and market technologies.

[2] Greenwood, Seshadri and Yorukoglu (2005) argue that the diffusion of appliances led to a fall in time spent in home production. However, Jones, Manuelli and McGrattan (2015) show that the above result only occurs if the elasticity of substitution between labor and capital in the home production function is sufficiently high.

the market), and household capital (durable goods in the form of residential structures and equipment used by households for home production). The crucial difference between the production in the sharing economy and in the standard market economy lies precisely in this distinction between business and household capital, since the difference between market and collaborative labor would be less accurate.[3]

The accumulation of domestic capital is another key element when analyzing the collaborative sector. A part of the goods and services produced by households is consumed by themselves at home and is what we refer to as domestic production. These are non-market goods and services, which are not intended for sale and which represent self-consumption. To produce these goods and services at home, individuals use the available stock of capital in these households, which is comprised of consumption in durable goods, and part of their available time. From this point of view, household spending on durable goods can be considered as investment spending in domestic capital assets, which range from refrigerators, ovens, dishwashers, washing machines and vehicles, to housing. The household's endowment of durable goods has a market value as they can be used as productive capital. These durable goods can be classified as shareable goods, since they can generate services that do not have to be fully and compulsorily consumed by household members, but can also be rented to other consumers. In addition, since households can devote some time to working in the production of goods and services within the collaborative sector, we define household time use as a tetrachotomy for the available discretional time when adding work time, in the sharing economy, to the uses of time considered in standard models.[4]

We consider an economy with three production sectors: the market production sector composed by firms, the household production sector producing a nontradable consumption good, and the production within the collaborative economy sector composed of households producing shareable goods and services. To avoid confusion, the market will refer to all goods and services produced by firms and purchased by households but excluding the collaborative consumption producing sector.

The main differences across the sectors arise from the inputs used in the production. Durable goods are used for both home production and collaborative consumption. In this model economy discretionary time is decomposed in four parts: working time, home work, time allocated in the sharing sector (sharing labor), and leisure. Home work activities

[3]Both types of labor may be interpreted as time devoted to produce goods and services which are marketed.

[4]Coyle (2016) suggests that people providing services in the UK sharing economy would account for around 3% of the UK's total workforce.

refer basically to meals, child care, laundry and cleaning, etc.[5] Collaborative consumption refers, for example, to car-sharing. Total consumption is a composite of market goods and services, shared goods and services, and home production, which are assumed to be imperfect substitutes. The production technology is assumed to be Cobb-Douglas in the three sectors. Total output is assumed to be the sum of market production plus the sharing economy production.

2.2 Households

In our economy, households not only take consumption and investment decisions, but they also decide between two possible outlets for the goods they produce using household inputs. Specifically, households produce two types of goods: home nontradable goods and services to be consumed by themselves, and tradable (shareable) goods and services to be marketed for consumption by other households. Investment is distributed between physical productive capital (structures and equipment) to be used by firms, and investment in durables that accumulates into home capital stock. Accordingly, we assume that they have access to two different production technologies, one for each production activity. The two production activities carried out by the households use a portion of the household capital stock.

We assume that the economy is inhabited by a stand-in representative consumer with the following instantaneous utility function:

$$U(C_t, L_t) = \gamma \log(C_t) + (1-\gamma)\log(1-L_t) \tag{1}$$

where C_t is total consumption of nondurable goods and services, L_t is non-leisure time (time used in either the market sector, the home production sector, or the collaborative consumption sector), and the preference parameter γ $(0 < \gamma < 1)$ is the proportion of consumption to total income. Total consumption is composed of consumption of goods and services purchased either from the market or from the collaborative consumption sector (denoted by the subindex z), and self consumption of home production (denoted by the subindex h). It is assumed that total consumption is given by a CES type aggregation function such as:

$$C_t = \left[\omega C_{z,t}^{\eta} + (1-\omega) C_{h,t}^{\eta}\right]^{1/\eta} \tag{2}$$

[5] As is standard in the literature, discretionary time is defined as total time less sleeping and personal care time. Some authors, such as Greenwood and Hercowitz (1991), include time devoted to home production into leisure. However, as is pointed out by Benhabid, Rogerson and Wright (1991) it would be more correct to differentiate between time spent in home production which generates disutility from time spent in leisure, which generates utility. In our framework, we divide discretionary time into four components: market work, sharing economy work, home work, and leisure.

where $C_{z,t}$ is the consumption of nondurable goods and services purchased, and $C_{h,t}$ is the consumption of home production, where η is the parameter measuring the willingness of agents to substitute between the two goods, and ω $(0 < \omega < 1)$ is the proportion of each type of goods in the total consumption. The parameter η will be key for the relationship between home activities and the rest of the activities. The elasticity of substitution between the consumption of purchased goods and services and the consumption of home production is defined as $1/(1-\eta)$. If η is equal to 1, then both types of goods are perfect substitutes and total consumption is the same for each type of consumption. On the other hand, if $\eta = 0$, total consumption would be a Cobb-Douglas function of both types of goods and the elasticity of substitution would be unitary.

Consumption of nondurable goods and services purchased in turn is an aggregate of nondurable goods and services purchased from the market, $C_{m,t}$, and from the collaborative consumption, $C_{s,t}$, that is, consumption of rented goods and services in the sharing economy:

$$C_{z,t} = \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{1/\upsilon} \tag{3}$$

where υ is the parameter measuring the willingness of agents to substitute between the two goods, and μ $(0 < \mu < 1)$ is the proportion of each good in the total purchased consumption. In principle, one would expect that the elasticity of substitution between market and shareable goods and services is larger than between the combination of these two goods and the goods and services produced and consumed at home. This would imply that $\eta < \upsilon$.

Total available effective time endowment of the economy is normalized to 1, and is defined as the non-sleeping hours of the working-age population. Each household can employ this endowment of time in three different activities (apart from leisure): market goods production ($L_{m,t}$), home work ($L_{h,t}$), and sharing labor ($L_{s,t}$). Henceforth, leisure is defined as the residual $1 - L_t$, whereas non-leisure time, L_t, is given by:

$$L_t = L_{m,t} + L_{h,t} + L_{s,t} \tag{4}$$

Therefore, the households' instantaneous utility function can be defined as:

$$U(C_{m,t}, C_{s,t}, C_{h,t}, L_{m,t}, L_{s,t}, L_{h,t}) \tag{5}$$

$$= \gamma \log \left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]^{1/\eta} \tag{6}$$

$$+(1-\gamma)\log(1 - L_{m,t} - L_{s,t} - L_{h,t}) \tag{7}$$

Households' budget constraint is defined as:

$$C_{m,t} + C_{s,t} + I_{k,t} + I_{d,t} = W_{m,t} L_{m,t} + R_{m,t} K_t + W_{s,t} L_{s,t} + R_{s,t}(1-\chi) D_t \tag{8}$$

where $I_{k,t}$ is investment in market physical capital (this capital will be rent to the firms), $I_{d,t}$ investment in durable goods, $W_{m,t}$ is the wage in the market, $R_{m,t}$ is the rental price of market capital, K_t is the market physical capital stock, $W_{s,t}$ is the wage in the sharing economy sector. The last term on the right-hand side of equation (8) follows from our characterization of the sharing economy, where the home capital stock, D_t, which is composed of a combination of durable goods and residential structures now has two alternative uses: a fraction is used for home production, χ (where $0 < \chi < 1$), and the other fraction, $(1-\chi)$, is rented in the collaborative economy marketplace at rate $R_{s,t}$ (rental price of home capital used in the collaborative economy production).

Notice that our specification considers that the investment decision on durables to be used in domestic production is not independent from the one to be used in the sharing economy sector. This introduces an additional difference between the market economy sector and the sharing economy sector, as households do not split the investment decision on durables into two parts. In other words, households decide how much home capital stock they want to accumulate but not how much home capital stock they want to accumulate to be used in sharing economy production and how much in home production.

Household capital, D_t , accumulates according to

$$D_{t+1} = (1-\delta_d)D_t + Q_{d,t}I_{d,t} \tag{9}$$

where $0 < \delta_d < 1$ is the home capital depreciation rate, and $Q_{d,t}$ is the investment-specific technological change to home capital.

Similarly, market physical capital stock evolves as:

$$K_{t+1} = (1-\delta_k)K_t + Q_{k,t}I_{k,t} \tag{10}$$

where $0 < \delta_k < 1$ is the physical market capital depreciation rate, and $Q_{k,t}$ is the investment-specific technological change to market capital.

Following Greenwood, Hercowitz and Krusell (1997, 2000), we assume that ISTCs to both market capital and home capital follow an exogenous autorregressive process.

The household chooses the sequences of $\{C_{m,t}, C_{h,t}, C_{s,t}, L_{m,t}, L_{h,t}, L_{s,t}, I_{k,t}, I_{d,t}\}$ so as to solve the following maximization problem:

$$\underset{\{C_{m,t},C_{h,t},C_{s,t},L_{m,t},L_{h,t},L_{s,t},K_t,D_t\}_t^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_t + (1-\gamma)\log(1-L_{m,t}-L_{h,t}-L_{s,t})\right] \tag{11}$$

subject to the budget constraint and the technological constraint for home production to be defined later, given the initial market capital stock K_0, the initial stock of durables, D_0, and where $\beta \in (0,1)$, is the discount factor.

2.3 The market sector

The goods and services market sector is formed by firms that represent the standard market sector. In the market sector the firm has the problem of finding optimal values for the utilization of market labor and market capital. The market production function of the stand-in firm, $Y_{m,t}$, is assumed to be a standard Cobb-Douglas specification of market labor services, $L_{m,t}$, and market capital, K_t, with constant returns to scale:

$$Y_{m,t} = A_t K_t^{\alpha} L_{m,t}^{1-\alpha} \tag{12}$$

where A_t is a measure of total-factor productivity, and $0 < \alpha < 1$ is the private capital share of output. Goods and factors markets are assumed to be perfectly competitive. The firm rents market capital and hires labor to maximize period profits, taking both goods' and factors' prices as given.

The firms decision problem can be defined as a static maximization problem:

$$\max_{K_t, L_t} \left[A_t K_t^{\alpha} L_{m,t}^{1-\alpha} - R_{m,t} K_t - W_{m,t} L_{m,t} \right] \tag{13}$$

First order conditions are given by:

$$\frac{\partial \Pi_t}{\partial K_t} : \quad R_{m,t} - \alpha A_t K_t^{\alpha-1} L_{m,t}^{1-\alpha} = 0 \tag{14}$$

$$\frac{\partial \Pi_t}{\partial L_t} : \quad W_{m,t} - (1-\alpha) A_t K_t^{\alpha} L_{m,t}^{-\alpha} = 0 \tag{15}$$

Notice that this definition of the market sector does not include all tradeable production, nor the total inputs, as the collaborative consumption sector is excluded.

2.4 Home production sector

Following Reid's (1934) concept, home production is defined as "those unpaid activities with are carried on, by and for the members, which activities might be replaced by market goods, or paid services, if circumstances such as income, market conditions, and personal inclinations permit the service being delegated to someone outside the household group". Household production activities are not included activities in the economy, since they refer to activities not traded in the market, that is, they do not have market prices.

We consider that households can use time and home capital to produce a nontradable consumption good for self-consumption. It is assumed that the home production function is a Cobb-Douglas type with constant returns to scale:

$$H_t = B_t (\chi D_t)^{\theta} L_{h,t}^{1-\theta} \tag{16}$$

where B_t is a measure of home work productivity shock, and $0 < \theta < 1$ is the technological parameter representing the elasticity of home production with respect to the household capital (χD_t). That is, we assume that part of the home capital is involved in the home production. McGrattan *et al.* (1997) consider a similar approach when using a CES function for home production technology, and a fraction of total capital is used in the home production sector. Thus, we follow Greenwood and Hercowitz (1991) and we identify household capital with durable goods.

Furthermore, home production is only used for household consumption, such that,

$$H_t = C_{h,t} \tag{17}$$

2.5 Sharing economy production sector

In this theoretical context, the collaborative economy arises when households use their endowment of productive factors, either time or home capital, in the production of goods and services to be provided to other individuals. In this sense, it would not be differentiated from the other existing productive market activities, since it consists of a rental of productive factors. However, it can be differentiated by the fact that it makes productive factors that are originally only used in domestic production available to other households. Therefore, it would be a sector that shares characteristics of both the standard productive sector (for the goods and services it offers) and the domestic productive sector (due to the productive factors it uses).

Here, we describe the technology in the sharing sector. We assume that households have access to a technology for producing tradable goods by using additional time and household capital inputs. The sharing economy sector mixes characteristics of both the market and home production sectors. This sector produces goods and services for the sharing marketplace that compete with those goods and services produced by firms as they are close substitutes. Second, households can decide to allocate time for the production of goods and services in the collaborative economy. This labor time, called sharing labor and denoted by $L_{s,t}$, has similar characteristics to the market labor. We assume that this sharing labor is paid a wage equal to its productivity. Third, the capital input used in the sharing production function is a portion of the stock of home capital (durables) denoted by $(1-\chi)D_t$. Thus, the stock of home capital can be used either as capital input in the production for self consumption or in the production of goods and services destined for the collaborative economy.

Households can decide to use non-market inputs to produce goods and services that can be rented in the collaborative economy marketplace. We assume a standard Cobb-

Douglas function with constant-returns-to-scale to represent the production in the sharing economy sector, S_t, as follows:

$$S_t = Z_t((1-\chi)D_t)^{\phi} L_{s,t}^{1-\phi} \tag{18}$$

where Z_t is a productivity shock specific to this sector, and ϕ is the technological parameter representing the elasticity of the sharing economy output with respect to the proportion of household capital used in this sector $((1-\chi)D_t)$.

We assume that the household decision problem from their role as a producer participating in the collaborative consumption sector consists of the maximization of the following profit funciton,

$$\max_{D_t, L_{s,t}} \left[Z_t((1-\chi)D_{s,t})^{\phi} L_{s,t}^{1-\phi} - R_{s,t}(1-\chi)D_t - W_{s,t}L_{s,t} \right] \tag{19}$$

First order conditions are given by:

$$\frac{\partial \Pi_{s,t}}{\partial D_t} \; : \; R_{s,t} - \phi Z_t((1-\chi)D_t)^{\phi-1} L_{s,t}^{1-\phi} = 0 \tag{20}$$

$$\frac{\partial \Pi_{s,t}}{\partial L_{s,t}} \; : \; W_{s,t} - (1-\phi) Z_t((1-\chi)D_t)^{\phi} L_{s,t}^{-\phi} = 0 \tag{21}$$

We will assume that the sharing economy sector cannot produce investment goods. Investment goods, both business capital and durables, can only be produced by the standard market economy. This is equivalent to assuming that production in the sharing economy is more related to services than to goods. To account for this asymmetry in the type of goods produced by the market sector versus the sharing economy sector, the model considers that production in the sharing sector is equal to the consumption of these goods, and hence, $C_{s,t} = S_t$.

2.6 Household's maximization problem

The Lagrangian auxiliary function associated with the households' maximization problem is defined as:

$$\mathcal{L} = \beta^t \begin{bmatrix} \gamma \log \left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon} \right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta} \right]^{1/\eta} \\ +(1-\gamma)\log(1 - L_{m,t} - L_{s,t} - L_{h,t}) \end{bmatrix} \tag{22}$$

$$-\lambda_{1,t} \begin{bmatrix} C_{m,t} + C_{s,t} + \frac{K_{t+1} - (1-\delta_k)K_t - Q_{k,t}R_{m,t}K_t}{Q_{k,t}} \\ + \frac{D_{t+1} - (1-\delta_d)D_t - Q_{d,t}R_{s,t}(1-\chi)D_t}{Q_{d,t}} \\ -W_{m,t}L_{m,t} - W_{s,t}L_{s,t} \end{bmatrix} \tag{23}$$

$$-\lambda_{2,t}[C_{h,t} - B_t(\chi D_t)^{\theta} L_{h,t}^{1-\theta}]$$

From first order conditions we obtain the following values for the Lagrangian multipliers:

$$\lambda_{1,t} = \frac{\beta^t \gamma\mu\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{m,t}^{\upsilon-1}}{\left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]} \tag{24}$$

$$\lambda_{2,t} = \frac{\beta^t \gamma(1-\omega)C_{h,t}^{\eta-1}}{\left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]} \tag{25}$$

In equilibrium, the relationship between consumption of market goods and services and shareable goods and services is given by:

$$\mu C_{m,t}^{\upsilon-1} = (1-\mu)C_{s,t}^{\upsilon-1} \tag{26}$$

Equilibrium allocation time is given by the following conditions where the wage in the standard productive market sector is equal to the wage that prevails in the sharing economy sector,

$$\frac{(1-\gamma)}{(1-L_{m,t}-L_{s,t}-L_{h,t})} = \gamma(1-\omega)\left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]^{(1-\eta)/\eta} C_{h,t}^{\eta-1}(1-\theta)B_t D_{h,t}^{\theta} L_{h,t}^{-\theta} \tag{27}$$

$$\frac{(1-\gamma)}{(1-L_{m,t}-L_{s,t}-L_{h,t})} = \gamma\omega\mu\left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]^{(1-\eta)/\eta} \left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{m,t}^{\upsilon-1} W_{m,t} \tag{28}$$

$$\frac{(1-\gamma)}{(1-L_{m,t}-L_{s,t}-L_{h,t})} = \gamma\omega(1-\mu)\left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]^{(1-\eta)/\eta} \left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{m,t}^{\upsilon-1} W_{s,t} \tag{29}$$

Equilibrium investment decisions in market physical capital and home capital are given by

$$\frac{\lambda_{1,t}}{Q_{k,t}} = \lambda_{1,t+1}\left[\frac{(1-\delta_k) - Q_{k,t+1}R_{m,t+1}}{Q_{k,t+1}}\right] \tag{30}$$

$$\frac{\lambda_{1,t}}{Q_{d,t}} = \lambda_{1,t+1}\left[\frac{(1-\delta_d) + Q_{d,t+1}(1-\chi)R_{s,t+1}}{Q_{d,t+1}}\right] + \lambda_{2,t+1}\theta\frac{C_{h,t+1}}{D_{t+1}} \tag{31}$$

Fisher (2007) develops a home production model with household capital complementarity with business inputs, by assuming that the supply of effective hours is derived from a production function that uses the market time and the household capital as inputs.

When complementarity is present, household capital also contributes to the production of market goods (additional to home production goods), as an additional capital input to business capital. This introduces an additional incentive for households to accumulate household capital. In our theoretical framework, we also found an additional incentive for home capital accumulation, as this capital contributes to the production of both nontradable home goods for self consumption and tradable goods for collaborative consumption.

The model incorporates five technological shocks: three productivity shocks specific to each production sector, and two ISTC shocks for business capital and household capital. We assume that aggregate productivity shocks to each production sector are not related with ISTCs to capital assets. Aggregate productivity shocks to the three production sectors are governed by the following vector AR(1) process:

$$\begin{bmatrix} \log A_t \\ \log B_t \\ \log Z_t \end{bmatrix} = \begin{bmatrix} \rho_A & \zeta_{AB} & \zeta_{AZ} \\ \zeta_{BA} & \rho_B & \zeta_{BZ} \\ \zeta_{ZA} & \zeta_{ZB} & \rho_Z \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log B_{t-1} \\ \log Z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^A \\ \varepsilon_t^B \\ \varepsilon_t^Z \end{bmatrix} \tag{32}$$

where $\rho_i < 1$, ε_t^i is a random perturbation and ζ_{ij} represents the correlation among shocks.

Similarly, for ISTC shocks, we also consider the following vector AR(1) process:

$$\begin{bmatrix} \log Q_{k,t} \\ \log Q_{d,t} \end{bmatrix} = \begin{bmatrix} \rho_{Qk} & \zeta_{Qkd} \\ \zeta_{Qdk} & \rho_{Qd} \end{bmatrix} \begin{bmatrix} \log Q_{k,t-1} \\ \log Q_{d,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{Qk} \\ \varepsilon_t^{Qd} \end{bmatrix} \tag{33}$$

We study two cases. In the first one, we assume that technology shocks are idiosyncratic to each sector and there is no transmission of shocks from one sector to the others, that is, $\zeta_{ij} = 0$. In the second case, we assume that innovations to productivity or to the investment spill over into the other sectors. Greenwood and Hercowitz (1991) argue that the composition of structures and equipment in business capital is similar to that of household capital. Under this assumption ISTC shocks to business capital should be perfectly correlated with ISTC shocks to household capital. Only results for the first case are presented below.

3 Calibration and the steady state

To calibrate the model, values must be assigned to the following set of parameters:

$$\Omega = \{\alpha, \theta, \phi, \delta_k, \delta_d, \beta, \gamma, \eta, \omega, \upsilon, \mu, \chi\}$$

and to the parameters driving the exogenous technological processes. Parameter values for the market production sector are taken from the literature. Parameter values for the home production sector are taken from previous calibration and/or estimation of DSGE models with a home production sector. Finally, parameter values for the sharing economy are new, as no previous calibration and/or estimation of a model incorporating this sector have been done in the literature yet. Some parameters are chosen to match some steady state values.

Table 1 shows the calibrated parameters to be used in the simulation of the model and chosen to match some steady state values. We use annual data. The discount factor has been fixed at 0.960 so that the real interest rate corresponds to a 4% yearly interest rate. The preference parameter representing the weight of consumption in the utility function is chosen to be 0.55. Rupert, Rogerson and Wright (1995) use micro data to try to determine preferences and technological factors that affect the production and consumption of home produced goods. Benhabid *et al.* (1991), using data from the Panel Study of Income Dynamics, estimated a value of η =0.6 for the parameter governing the elasticity of substitution between market and home produced goods. However, in calibrating their model, they used a value of $\eta = 0.8$ as this is the average between their estimated value and the estimation by Eichenbaum and Hansen (1990).[6] McGrattan *et al.* (1997) estimated values of $\omega = 0.414$ and $\eta = 0.429$. Here we we assume that $\eta = 0.429$, following McGrattan *et al.* (1997), and $\omega = 0.5$, as collaborative consumption goods are also included in the market goods and services composite. Baxter and Jermann (1999) use a value of 0.63 for the share of tradable goods in the consumption aggregate. No information is available regarding the sharing economy sector parameters. For preference parameters, we chose a larger value for the elasticity of substitution between market and sharing economy goods than for trade goods versus home nontraded goods. For our benchmark calibration, preference parameters for the CES function of the aggregate of market and sharing consumptions are fixed at $\upsilon = 0.2$, and $\mu = 0.90$.

According to the literature, the capital share in the market production function is fixed at $\alpha = 0.35$. The capital share in the home production function is set at $\theta = 0.08$ by Benhabid *et al.* (1991). Fisher (2007) uses values of 0.19 for the model with household capital complementarity and a value of $\theta = 0.30$ for the standard model with home production. This value was chosen to produce a steady state home consumption over output of 0.26. This implies a steady state durable to market capital ratio of only

[6]Eichenbaum and Hansen (1990) used aggregate data to estimate a model in which households value the services from market consumption goods and durables goods. Under the assumption that the home production function only uses home capital (durables), the flow of services from durables can be equivalent to the consumption of home produced goods. They found little evidence againts the hypothesis that durable and non-durable goods are perfect substitutes ($\eta = 1$).

0.14, defining home capital as home equipment and furniture without including houses. Greenwood and Hercowitz (1991) use a CES technology for the home production sector, with a share parameter of household capital of 0.13 in a model with no leisure. Baxter and Jermann (1999) use a labor share of 0.80 in the household sector ($\theta = 0.20$). Here we follow Fisher (2007) and we use a value of $\theta = 0.30$. The technological parameter for the sharing economy production fuction (ϕ) has been fixed at 0.5, as it is assumed that this sector is more capital intensive.

Market capital depreciation rate and durables depreciation rate are assumed to be equal and fixed at 0.06. Fisher (2007) uses values, for quarterly data, of 0.019 for business capital depreciation and 0.017 for home capital depreciation. Greenwood and Hercowitz (1991) choose a value of δ_k =0.078 for business capital annual depreciation, a figure obtained from the average service life of nonresidential structures and equipment for the sample period 1954-1985. They assume that the composition of structures and equipment in business capital is similar to that of household capital and hence the same depreciation rate is assumed for household capital. Baxter and Jermann (1999) used values of 0.0255 for business capital and 0.0230 for household capital, on a quarterly basis.

The share of durables in home production versus the sharing economy sector has been calibrated internally using steady state expressions from the model. For the benchmark calibration the resulting share is $\chi = 0.938$, which implies that only 6.2% of total household capital is used for sharing economy production.

Tabla 1: Calibrated parameters

	Parameter	Definition	Value
Preferences	β	Discount factor	0.960
	γ	Consumption-leisure preference parameter	0.550
	η	Home goods substitution parameter	0.429
	ω	Consumption of tradable goods proportion	0.500
	υ	Sharing goods substitution parameter	0.200
	μ	Consumption of market goods proportion	0.900
Technology	α	Market technological parameter	0.350
	δ_k	Market capital depreciation rate	0.060
	θ	Home production capital share	0.300
	δ_d	Home capital depreciation rate	0.060
	ϕ	Collaborative consumption capital share	0.500
	χ	Share of durables used in home production	0.938

Finally, we assume that the parameters of the autoregressive process for productivity and ISTC in all three sectors are the same, that is, we will assume that $\rho = 0.95$ and

$\sigma = 0.01$, consistent with the literature. On the other hand, as indicated by Benhavid *et al.* (1991) correlation among shocks can be different from zero. Following Fisher (1997, 2007) and Greenwood, Hercowitz and Krusell (1997) the growth rates of investment technologies is higher for home capital (a gross growth rate of 1.003) than for business capital (a gross growth rate of 1.0024).

Table 2 shows the steady state values for the calibrated model economy. The steady state of our model economy can be used to quantify the sharing sector in the total economy given our benchmark calibration. Total output is defined as the sum of market output and the sharing economy output. Several aspects must be highlighted here. First, we find that the ratio of consumption of market goods to the market output is 77%, representing a saving rate of 23% at steady state while the capital/output ratio is 3.6. These values are exactly the same as those that would result in the model without the household sector, so its inclusion does not alter the steady state of the market sector of the economy. The steady state value for time devoted to home work by Benhabid *et al.* (1991) is 0.28. Hersch and Stratton (1994), using data from the Michigan Panel Study of Income Dynamics, for the period 1979-1982, found that total time spent on housework (white, married workers, aged 20-64) is on average 27.02 hours per week. Considering an available discretionary time of 16 hours per day, that figure results in a portion of time devoted to home work of 0.2412 for a seven-days week and 0.2815 for a six-days week. Benhabid *et al.* (1991) considered a fraction of 0.28 for home work and 0.33 for market work. Bridgman, Duernecker and Herrendorft (2018) collected data for a large number of countries, where total hours in both home production and market production vary between 40 and 65 hours, with home hours accounting for nearly half of the total working hours. Baxter and Jermann (1999) fixed a fraction of 0.25 spent in the housework sector. Second, we can observe what the distribution of time at steady state is for market working activities and home production activities. With the calibrated parameters we find that the proportion of time devoted to market working is 0.32, while the proportion of time spent on housework is 0.27, and only a portion of 0.01 is devoted to sharing economy working, with the remaining time (about 0.40) left for leisure. As expected, time spent on home activities is less than the time spent working in the market, but still represents a significant proportion of total available time. By contrast, working time in the sharing economy sector represents a very small proportion of total available time.

Total investment in steady state represents 36% of total output. This share is large because total investment includes both business capital investment and investment in durable goods. In fact, we obtain that the fraction of investment in business capital is 22% of total output, whereas the fraction of investment in durables is 14% of total output.

Fisher (2007) measures business capital as private nonresidential fixed capital, whereas household capital is measured to include private residential capital and consumer durables. Consumption is measured as nondurables and services, excluding housing services. Analogously, output is measured as GDP less consumption of housing services. Fisher obtained that, for the sample period 1948-2004, the average business capital-output ratio is 4.66, and that the average home capital-output ratio is 6.15. According to these figures, the average home capital over business capital ratio is 1.3197. He also obtained that the corresponding investment shares are 0.12 for business capital and 0.16 for home capital. Greenwood and Hercowitz (1991) obtain an average home-business capital stock of 1.13 for the period 1954-1988. In the model without capital taxation, the ratio reduces to 0.80. Baxter and Jermann (1999) set the ratio of household capital to market capital at 0.625, a share of market investment of 0.118 for business capital and a share of 0.0675 for household investment. Our values for the share of investment are larger than these figures and the household capital lower than business capital, consistent with the values of Baxter and Jermann (1999) but contrary to the figures of Greenwood and Hercowitz (1991) and Fisher (2007).

Table 2: Steady state values

Variable	Definition	Value	Ratio to $\overline{Y}$
$\overline{Y}$	Total output	0.7192	1.0000
$\overline{Y}_m$	Market output	0.6715	0.9337
$\overline{Y}_s$	Sharing economy output	0.0476	0.0662
$\overline{C}_m$	Market goods consumption	0.4130	0.5742
$\overline{C}_s$	Sharing economy goods consumption	0.0476	0.0662
$\overline{C}_h$	Home goods consumption	0.4608	-
$\overline{I}$	Total investment	0.2584	0.3593
$\overline{I}_k$	Investment in market capital	0.1551	0.2157
$\overline{I}_d$	Investment in durables	0.1034	0.1438
$\overline{K}$	Market physical capital stock	2.5848	3.5940
$\overline{D}$	Household capital (durables) stock	1.7232	2.3960
$\overline{L}_m$	Market labor	0.3249	-
$\overline{L}_s$	Sharing economy labor	0.0142	-
$\overline{L}_h$	Homework labor	0.2691	-

4 Sharing economy and productivity technological shocks

Recent expansion of the sharing economy is a by product of the technological change that allows households to directly participate in the market as a production unit by using household capital as an input along with additional working time (other than that used in the market). Therefore, it is important to consider how the dynamics of each sector react to technological factors. This section studies the dynamic effects of technological shocks on the model economy. The model developed above has three aggregate productivity shocks to each sector, and two investment-specific technological shocks for business capital and household capital. ISTC shocks to home capital will affect both the home production sector and the collaborative economy sector in the same way. In this section we explore how our simulated economy responds to each technological shock in order to investigate the relationship between the sharing economy sector and the two other production sectors of our model economy.

First, we study the response to an aggregate productivity shock (neutral technological shock) in the market sector. This is the standard shock studied for assessing the properties of the model in the business cycle and will be used to compare the response of our model economy with that of models without the sharing economy sector. The literature has studied the effects of this shock in a model with home production and it results in negative effects on the home production sector as households devote more resources (hours and investment) to the market sector (see, for example, Benhabid *et al.* 1991; Greenwood and Hercowitz, 1991). Here we extend that analysis to the sharing economy sector.

Figure 1 plots the impulse-response of the main variables in the economy to a positive aggregate productivity shock specific to the market sector. This positive productivity shock to the market sector, as expected, increases market production and business capital investment and consumption of market goods increases. As a consequence the amount of inputs, capital and labor, also increase in the market sector, which increases the persistence of the positive effects of the shock on market output. The impact of the shock on the home production sector is consistent with the literature. The standard model with home production produces a negative effect on the home production sector from a positive TFP shock to the market sector by reducing the stock of household capital (see Benhabid *et al.*, 1991). However, we observe that the negative impact on the investment in household capital (durable goods) turns out to be positive after some periods.

Focusing on the sharing economy sector, we found that the shock in the market sector reduces the number of hours devoted to production in the sharing economy sector.

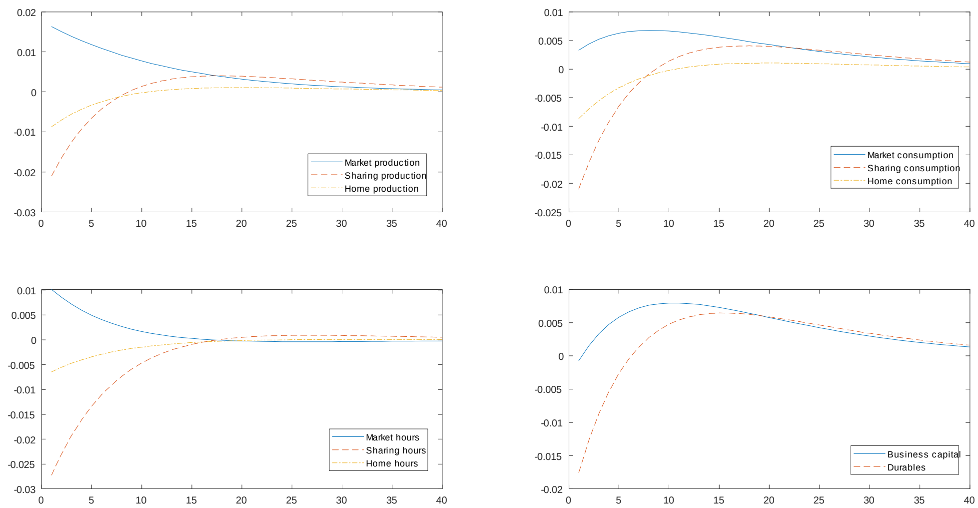

Figure 1: Impulse-response functions to a positive productivity shock in the market sector

This effect, in combination with the initial negative initial on household capital investment has a negative impact on sharing outuput. However, given the positive impact on household capital investment after some periods, both sharing output and consumption increases over their steady state values. The economic intuition behind these results is straightforward. Initially, the impact of the shock produces a reallocation of investment to the market sector, reducing investment in household capital. This initial effect leads to a negative effect on output in both home production and sharing sectors. However, the expansion of the economy increases effective consumption, leading to a recovery of spending in durable goods, increasing investment in household capital which in turns increases sharing production.

In sum, introducing the sharing economy into the model also has important implications for the relationship between the market sector and the sharing economy and the home production sector across the business cycle. Aggregate productivity shocks to the market sector spill over to the sharing economy, with a negative impact on home production. The consumption of goods and services marketed by both the market and the sharing economy sectors increases. The positive impact on the sharing economy is explained by the increase in the investment in durables after some periods. This means that both business capital and household capital (durable goods spending) are procyclical, as observed in the data. In this framework, hours increase in the market sector and decrease in the other two sectors, which is consistent with previous findings. Therefore, the in-

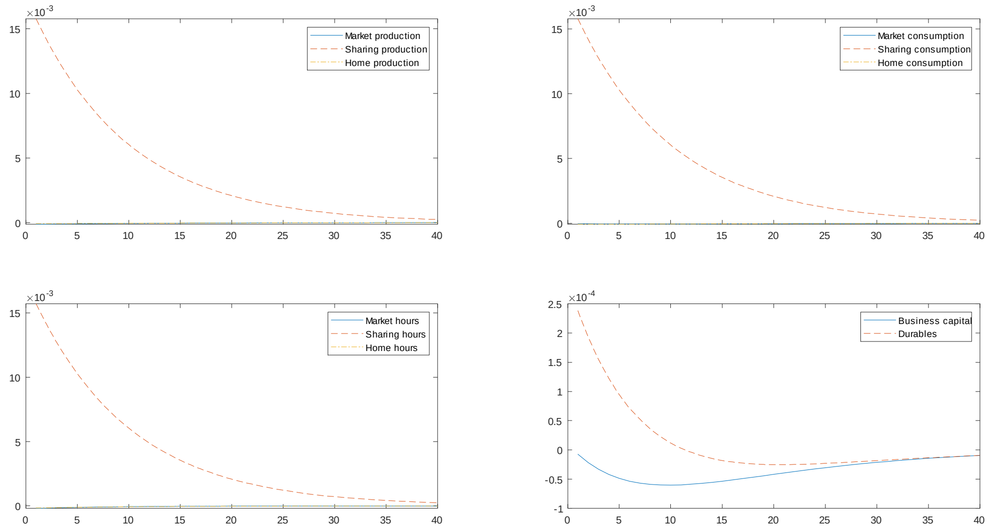

Figure 2: Impulse-response functions to a positive aggregate shock to the sharing economy sector

troduction of the sharing economy sector into the model incorporates a new transmission mechanism from an aggregate productivity shock to the market sector. This is because the sharing economy sector is also linked to the home production sector as they use the same capital input.

Second, an idiosyncratic neutral technological shock to the sharing economy sector is simulated. The expansion of the activities in the sharing economy have been fueled by the advances in ICTs, mainly, from the development of the Internet. Therefore, as a consequence of technical change associated with ICTs, positive productivity shocks are expected to be generated by technological advances in this sector, and hence, the study of how productivity shocks to this sector expand to the rest of the economy should be of interest.

Figure 2 plots the impulse-responses of the main variables to a positive aggregate productivity shock to the sharing economy sector. As expected, the shock increases total output in this sector, expanding collaborative consumption. This expansion in the sharing economy sector is fueled by an increase in hours devoted to this sector and in household capital. The impact of this productivity shock on the other two sectors is pint-sized. This is somewhat expected, given the relative small size of the sharing sector calibrated in the model economy with respect to the rest of the economy. Impulse-responses functions have been calculated as percentage deviation of each variable with respect to its steady

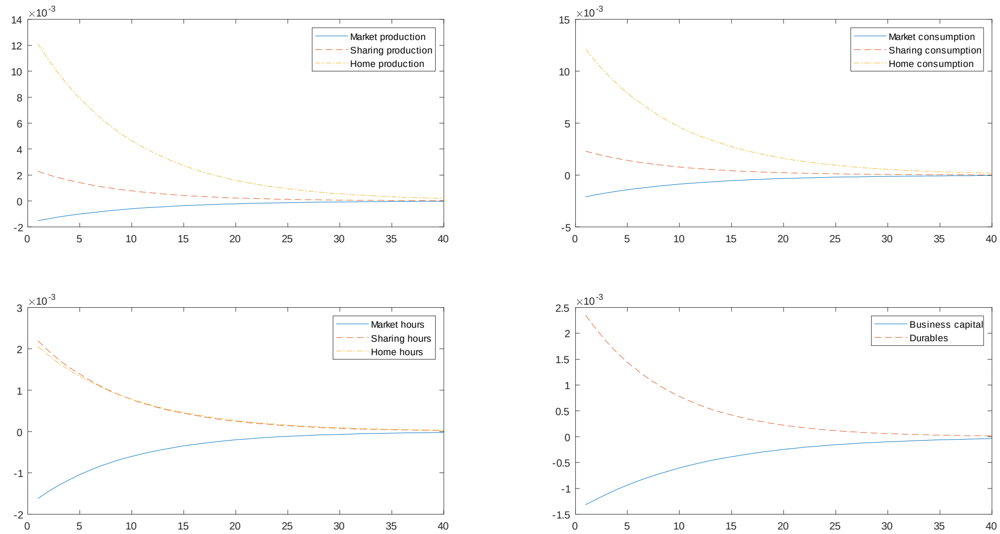

Figure 3: Impulse-response functions to a positive aggregate productivity shock in the home production sector

state. Therefore, the estimated impact of the aggregate productivity shock to the sharing economy has a very small impact on the other two sectors which respresent a very large fraction of the whole economy.

Third, we study the response of the economy to a shock in the home production sector. In the standard home production model this shock increases home production, which increases both home work hours and household capital stock, and reduces market output, by decreasing both labor and business capital stock. Our model produces similar results regarding both the home and the market sectors, but extending the analysis to the sharing economy sector, which is also positively affected by this shock.

Figure 3 plots the impulse-response functions corresponding to a positive productivity shock specific to the home production sector. As expected, the shock increases home production, which increases the number of hours used in this sector but also in the sharing economy, and reduces the number of hours worked in the market sector, also reducing investment in business capital. As a consequence, production in the market sector decreases but increases in the sharing sector. These results are consistent with those generated by standard home production models and we observe a negative impact on investment in business capital, while investment in durable goods rises. This introduces a link between productivity in the home production sector and the development of the sharing economy as both sectors use the same capital input.

In summary, these results indicate the existence of a positive link between the sharing economy and the other two sectors depending on the specific shock hitting the economy. We found that a positive productivity shock to the market economy has, over the time, a positive effect on investment in durable goods, in spite of an initial negative effect due to a reallocation effects of resources to the more efficient sector. On the other hand, we find also a positive link between a positive productivity shock to home production and the sharing economy due to the fact that both the sharing economy and home production technologies use the same capital input, durable goods, and thus, movements in the stock of durables affects both sectors in a similar way.

5 Sharing economy and ISTC shocks

Finally, the model economy includes two Investment-Specific Technological Change (ISTC) shocks for business capital and for durables. Whereas some capital assets are similar across the two sectors, other capital assets are specific either to the market capital aggregate or to the home capital aggregate. Therefore, both these ISTC shocks are not restricted to be the same for both types of capital. The question here is how the economy, and in particular the sharing economy sector, responds to an ISTC shock that is specific to each type of capital. We carry out two simulations. First, we study the cases of idiosyncratic ISTC shocks to each type of capital and no transmission of the shock from one type of capital to the other. Second, we assume that the composition of the two capital assets (equipment and structures) are equal and hence, correlation between the two shocks is one. This last case produces effects which are similar to the standard model with ISTC to equipment, but in our cases the effects extended to both the market and sharing economy sectors. Here, for this reason we focus on the first case.

Figure 4 plots the response of the economy to an ISTC shock specific to business capital. Total investment increases, as this shock increases investment in business capital. However, there is an initial adjustment in both types of investment, increasing it in business capital and reducing it in durables. As a consequence, the stock of business capital increases, but the stock of household capital first decreases, but then returns to positive figures after a period of time. Initially, the consumption decreases, as more resources are moved to investment. This negative consumption response to an ISTC shock has been previously documented by the literature (see Greenwood *et al.*, 2000). Therefore, we found that when the ISTC only affects business capital, the investment decision initially moves from durables to business capital, but after a period of time, the positive effects also translate to household capital accumulation. Hours decrease in both

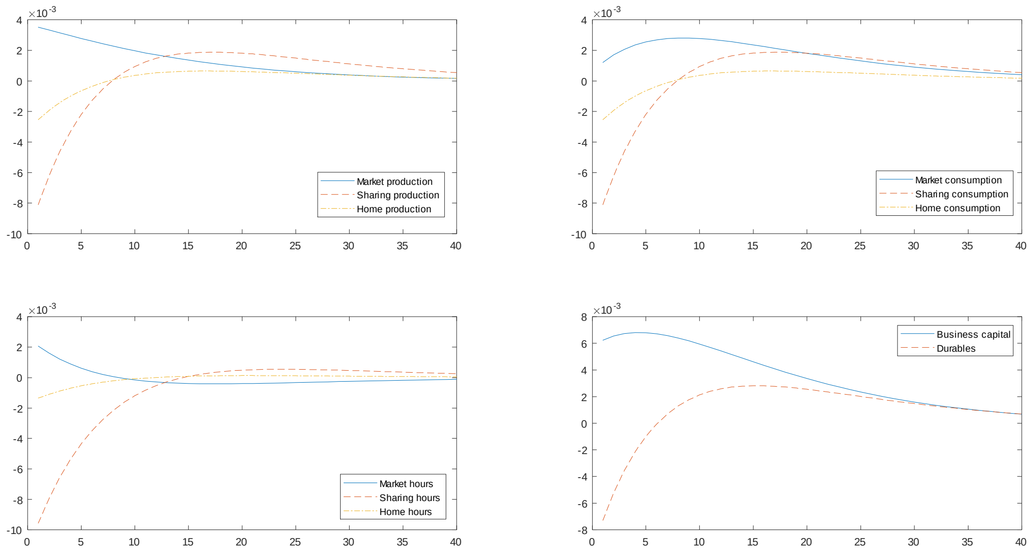

Figure 4: Impulse-response functions to a Investment-Specific Technological Shock to business capital

the sharing economy and the home production sectors, but increase in the market sector which eventually implies that leisure time is reduced.

ISTC shocks can also affect durable goods. Durable goods include home equipment and residential structures. Here, we assume that household capital assets can be different from business capital assets, and therefore, both types of capital are subject to specific ISTC shocks. These types of shocks have been studied in the literature using a model with home production. Jones, Manuelli and McGrattan (2015) studied the effects of an ISTC shock on household capital and found that the response of home work hours can be positive or negative, depending on the elasticity of substitution between market and home produced goods.

Figure 5 plots the impulse-response functions to an ISTC shock to durables. This shock will increase the stock of household capital, reducing the stock of business capital. As a consequence, output in both the home production sector and the sharing economy sector increases. A positive effect is found on these two sectors because each of them use household capital as an input. By contrast, market output decreases due to the decrease in the market hours worked and in business capital. All three types of consumption will increase, whereas the effect on total investment will be negative. This behavior in consumption is different from that observed when the ISTC shock affects business

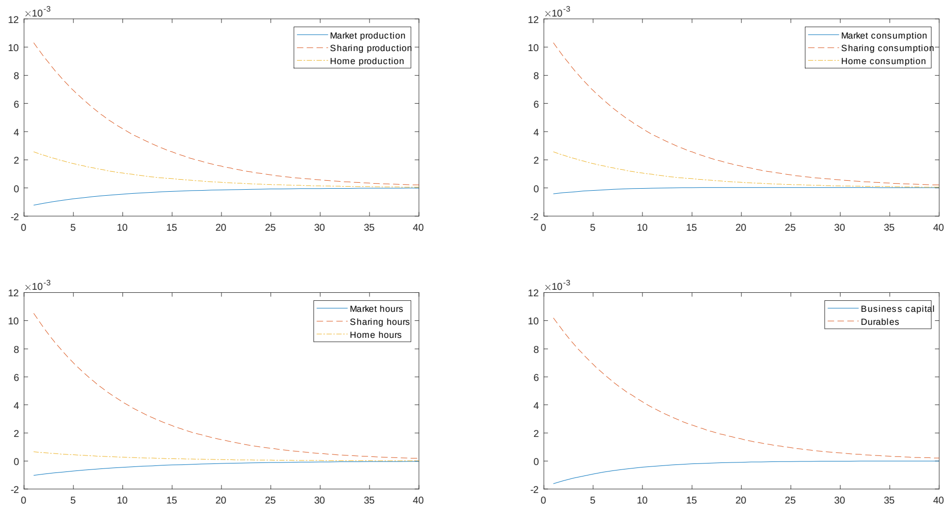

Figure 5: Impulse-response functions to a Investment-Specific Technological Change to household capital

capital, where a substitution effect of consumption by investment is observed. This is not the case when the ISTC shock is in household capital. Hours will increase not only in home production but also in the sharing economy sector, although the reduction in hours worked in the market leads to an increase in leisure time.

6 Conclusions

In the last decades, we have witnessed a resurgence of the collaborative economy. This renaissance is explained by technological progress and the accumulation of home capital in the form of durable goods. Technological progress and the development of Information and Communication Technologies (ICTs) have removed access market barriers and information constraints, enabling households to dedicate home production inputs which were only used previously for the production of home goods and services, to produce goods and services that are provided to other consumers. In light of the acknowledged importance of the sharing or collaborative economy for most advanced economies, it seems reasonable to come up with a new model that incorporates it as part of a general theoretical framework. This paper contributes to the literature by developing a DSGE model that includes the collaborative economy. More specifically, we introduce the collaborative economy into an otherwise standard model with home production. Thus, our model considers three production sectors: the market, the household and the collaborative sector.

We use a simulation of the model to stress the links among all three sectors. We found that productivity shocks to the market sector also have positive effects on the sharing economy sector by increasing investment in household capital. By contrast, productivity shocks to the sharing economy sector increase sharing consumption with little impact on the other two sectors given the relative small importance of the sharing economy on the whole economy. Finally, ISTC shocks on durables have a negative effect on market output, which increases production output in both home and sharing economy sectors and reduces business capital investment.

Although making a projection about the future is always risky, from our point of view, the collaborative economy sector will continue to expand, incorporating new goods and services in a gradual process which parallels technological progress. Specifically, despite housing being the greatest asset that households own in terms of value, many activities of the collaborative economy include the use of automobiles, personal computers, machinery and tools. In the near future, 3-D printers will most likely be included in this group which will mean that households will have access to the production of a large number of goods. In this sense, Petersen and Pearce (2017) make an interesting analysis on the implementation of 3-D printers in the home production of manufactured goods. According to this study, the development of this technology with the consequent cost reduction will allow the collaborative economy to expand to the industrial sector. Thus, it would be enough for a household to buy a 3-D printer and use it to produce basic industrial products that could be sold directly to other consumers.

Several extensions can be made. First, our model assumes that the sharing economy sector only produces services and non-durable goods. This assumption is introduced into our theoretical framework by equating the sharing economy production to the collaborative consumption, and the resulting consequence is that no investment is generated in this sector, in a similar fashion to the home production sector. However, this restriccion could be relaxed by considering the possibility that the sharing economy sector can also produce durable goods. Indeed, technological advances such as the 3-D printers referred to above, could be a fundamental transforming element in the sharing economy in the following years enabling the home production of durable goods. Second, our model assumes that the share of household capital used for home production and for sharing economy production is a constant. This assumption can be relaxed by considering the possibility that the investment decision in durables by households can be split in two parts, depending in which sector a particular durable good will be used for production, and allowing household capital mobility across the home production and the sharing economy sectors. These issues will be explored in future works.

Appendix

Here we show first order conditions for the household maximization problem. The maximization problem to be solved by the household is given:

$$\underset{\{C_{m,t},C_{h,t},C_{s,t},L_{m,t},L_{h,t},L_{s,t},K_t,D_t\}_t^\infty}{Max} \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_t + (1-\gamma)\log(1-L_{m,t}-L_{h,t}-L_{s,t})\right] \quad (34)$$

subject to the budget constraint,

$$C_{m,t} + C_{s,t} + I_{k,t} + I_{d,t} = W_{m,t}L_{m,t} + R_{m,t}K_t + W_{s,t}L_{s,t} + R_{s,t}(1-\chi)D_t \quad (35)$$

and the technological constraint for home production given by,

$$C_{h,t} = B_t(\chi D_t)^\theta L_{h,t}^{1-\theta} \quad (36)$$

given the initial market capital stock K_0, the initial stock of durables, D_0, and where $\beta \in (0,1)$, is the discount factor, where total consumption is defined as:

$$C_t = \left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]^{1/\eta} \quad (37)$$

By substituting investment in market capital and in home capital into the budget constraint, we have:

$$\begin{aligned} & C_{m,t} + C_{s,t} + \frac{K_{t+1}-(1-\delta_k)K_t}{Q_{k,t}} + \frac{D_{t+1}-(1-\delta_d)D_t}{Q_{d,t}} \\ = \; & W_{m,t}L_{m,t} + R_{m,t}K_t + W_{s,t}L_{s,t} + R_{s,t}(1-\chi)D_t \end{aligned} \quad (38)$$

The Lagrangian auxiliary function associated to the households' maximization problem is defined as:

$$\mathcal{L} = \beta^t \left[\begin{array}{c} \gamma \log\left[\omega\left[\mu C_{m,t}^{\upsilon} + (1-\mu)C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega)C_{h,t}^{\eta}\right]^{1/\eta} \\ +(1-\gamma)\log(1-L_{m,t}-L_{s,t}-L_{h,t}) \end{array} \right] \quad (39)$$

$$-\lambda_{1,t}\left[\begin{array}{c} C_{m,t} + C_{s,t} + \frac{K_{t+1}-(1-\delta_k)K_t - Q_{k,t}R_{m,t}K_t}{Q_{k,t}} \\ +\frac{D_{t+1}-(1-\delta_d)D_t - Q_{d,t}R_{s,t}(1-\chi)D_t}{Q_{d,t}} \\ -W_{m,t}L_{m,t} - W_{s,t}L_{s,t} \end{array} \right] \quad (40)$$

$$-\lambda_{2,t}[C_{h,t} - B_t(\chi D_t)^\theta L_{h,t}^{1-\theta}]$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial C_{m,t}} \quad : \quad \frac{\beta^t \gamma \mu \omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{m,t}^{\upsilon-1}}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} - \lambda_{1,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_{s,t}} \quad : \quad \frac{\beta^t \gamma (1-\mu) \omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{s,t}^{\upsilon-1}}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} - \lambda_{1,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} \quad : \quad -\frac{\lambda_{1,t}}{Q_{k,t}} + \lambda_{1,t+1} \left[\frac{(1-\delta_k) + Q_{k,t+1} R_{m,t+1}}{Q_{k,t+1}}\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial D_{t+1}} \quad : \quad -\frac{\lambda_{1,t}}{Q_{d,t}} + \lambda_{1,t+1} \left[\frac{(1-\delta_d) + Q_{d,t+1}(1-\chi) R_{s,t+1}}{Q_{d,t+1}}\right] + \lambda_{2,t+1} \theta \chi B_{t+1} (\chi D_{t+1})^{\theta-1} L_{h,t+1}^{1-\theta} = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_{h,t}} \quad : \quad \frac{\beta^t \gamma (1-\omega) C_{h,t}^{\eta-1}}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} - \lambda_{2,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_{m,t}} \quad : \quad -\frac{\beta^t (1-\gamma)}{(1 - L_{m,t} - L_{s,t} - L_{h,t})} + \lambda_{1,t} W_{m,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_{s,t}} \quad : \quad -\frac{\beta^t (1-\gamma)}{(1 - L_{m,t} - L_{s,t} - L_{h,t})} + \lambda_{1,t} W_{s,t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_{h,t}} \quad : \quad -\frac{\beta^t (1-\gamma)}{(1 - L_{m,t} - L_{s,t} - L_{h,t})} + \lambda_{2,t} (1-\theta) B_t (\chi D_t)^{\theta} L_{h,t}^{-\theta} = 0$$

For the first order conditions we obtain the following values for the Lagrange multipliers:

$$\lambda_{1,t} = \frac{\beta^t \gamma \mu \omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{m,t}^{\upsilon-1}}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} \tag{41}$$

$$\lambda_{2,t} = \frac{\beta^t \gamma (1-\omega) C_{h,t}^{\eta-1}}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} \tag{42}$$

In equilibrium, the relationship between consumption of market goods and services and those produced within the sharing economy is given by:

$$\mu C_{m,t}^{\upsilon-1} = (1-\mu) C_{s,t}^{\upsilon-1} \tag{43}$$

Equilibrium allocation time, is given by the following conditions where the wage in the standard market sector is equal to the wage in the sharing economy sector,

$$\frac{(1-\gamma)}{(1 - L_{m,t} - L_{s,t} - L_{h,t})} = \frac{\gamma (1-\omega)(1-\theta) C_t}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} \frac{C_{h,t}^{\eta}}{L_{h,t}}$$

$$\frac{(1-\gamma)}{(1 - L_{m,t} - L_{s,t} - L_{h,t})} = \frac{\gamma \omega C_t}{\left[\omega \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{\eta/\upsilon} + (1-\omega) C_{h,t}^{\eta}\right]} \left[\mu C_{m,t}^{\upsilon} + (1-\mu) C_{s,t}^{\upsilon}\right]^{(\eta-\upsilon)/\upsilon} C_{m,t}^{\upsilon-1} W_{m,t} \tag{44}$$

Equilibrium investment decisions in market physical capital and home capital are given by

$$\frac{\lambda_{1,t}}{Q_{k,t}} = \lambda_{1,t+1}\left[\frac{(1-\delta_k)+Q_{k,t+1}R_{m,t+1}}{Q_{k,t+t}}\right] \tag{45}$$

$$\frac{\lambda_{1,t}}{Q_{d,t}} = \lambda_{1,t+1}\left[\frac{(1-\delta_d)+Q_{d,t+1}(1-\chi)R_{s,t+1}}{Q_{d,t+1}}\right] + \lambda_{2,t+1}\theta\frac{C_{h,t+1}}{D_{t+1}} \tag{46}$$

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