Estimating the Constant Elasticity of Substitution Function of Rice Production.The case of Vietnam in 2012.

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Abstract

Vietnamese rice production has achieved remarkable success over last decades. By the land and market reforms, known as "Doi Moi", in which there were noticeable changes in policies such as land and production system were transformed from collective to individual contract system in 1980s, the process of legally privatization of farm properties, huge investment in irrigation system, Vietnam made progress in rice production. The country not only ensured its domestic demand but also started exporting rice and gradually became the second largest exporter in the world. An estimate of the constant elasticity of substitution function (CES) for Vietnam's rice production is essential for the government to design effective policy on agricultural production. This study makes the first attempt to estimate the nested CES model for Vietnam rice production in 2012. The paper finds that the elasticity of substitution of Vietnam's nested CES model lies between 0.44 to 0.46. The results indicate that the weak substitutability between land and the nest (labor, capital) in the nested CES model. The paper also provides empirical evidence that the nested CES structure in which capital with land are nested inputs and labor plays a role as the third input is rejected. This suggests that it is impossible to take labor as the substitutable factor for land and capital. The findings would partly contribute to design the Vietnam's effective policies on rice production with the appropriate allocation of inputs factors in order to achieve the optimal output.

JEL Classification: O12, O13, Q50, D58, C51

Keywords: Constant Elasticity of substitution, Levenberg-Marquardt method, Vietnam, rice production.

1 Introduction

In Vietnamese agricultural sector, rice production is considered as the most important industry since it plays a specially vital role in agricultural and rural economic development. Rice is an absolutely indispensable source of nutrition to millions of Vietnamese people and rice production contributes noticeably in ensuring national food security. On the other hand, the rice sector

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generates jobs for around more than two-third of labor force in Vietnam rural areas with the average contribution of 67 percent of household earning (ADB 2012, p.26). In addition, rice industry has important contribution to the programs of rural poverty alleviation, the country's gross domestic products (GDP) as well as being a considerable source of export revenue, which is roughly one fifth of the total export value (ADB 2012, p.20).

Vietnam has natural advantages for growing rice. Being a tropical country with high degree of humidity, annual plentiful rainfall and an intensive system of rivers and canals, Vietnam has fairly temperate weather and the soil is annually fortified by alluvium. With the advantageous conditions, rice is cultivated in almost regions across the country, among those, Red River Delta (RRD) in the North and Mekong River Delta (MRD) in the South are the two biggest granaries of Vietnam. The MRD with rice- planted areas of 2.1 hectares and 11.7 million people participated in the production is located around the Mekong River valley. Thus, the soil is fertile due to being raised by silt year by year, that allows to cultivate three rice crops in one year, those are the Winter- Spring crop from March to May, the Summer-Autumn crop from August to September and the Winter crop from December to January of the following year. With the area of 0.6 million hectares for growing rice and the labor force in rice production of 10.6 million, the RRD even has a better irrigation system in comparison with the MRD but this region has to face with severe weather conditions such as serious flood, drought which leads to generally two main crops per year: the Winter-Spring crop between May and June and the Winter crop between November and December. The two above main rice- grown areas together make up to 70 percent of the total rice yield (Kompas *et al*. 2012, p.483).

Vietnamese rice production has achieved remarkable success over last decades. Before 1980, the rice production could not meet the consumption demand within the country, therefore, the country had to import rice with the import amount up to 1.5 million ton during 1970s (GSO, 1995). However, surprisingly, by extensive land and market reforms, known as "Doi Moi", in which there were noticeable changes in policies such as land and production system were transformed from collective to individual contract system in 1980s (Mc Caig and Pavcnik 2013, p.24), the process of legally privatization of farm properties (Martin 2002, p.12), huge investment in irrigation system, etc…Vietnam made progress in rice production (Kompas *et al.* 2012, p.486). The country not only ensured its domestic demand but also started exporting rice and gradually became one of the most important rice exporters in the world. Currently, Vietnam is ranked the second biggest rice exporter, after only Thailand. In 2012, Vietnam's rice exports reached a record of 7.7 million tons with the total estimated value of USD 3.5 billion (GSO 2014). Thanks to intensive cultivation and multi-cropping as well as developing new and high quality hybrid rice seed, enhancing the irrigation system and fertilizer usage, the rice yielded shows a remarkable increase over time. Although being affected by the process of industrialization and urbanization, the land for planting rice has steadily narrowed by 11 percent, from 4.5 million hectares in 1999 to approximately 4.0 hectares at present, the intensive cultivation area for rice still achieved the number of 7.75 million hectares in the year 2012 due to the growing of triple crops in a year (GSO 2014). In 2010, the total rice production output was roughly 44 million tons, an increase of 11.5 million tons compared to 32.5 million tons in 2000 (FAOSTAT 2014). Rice productivity increased from 42.4 quintal per hectare to 56 quintal per hectare in 2000 and 2012 respectively (ADB 2012, p.32).

Though along with those outstanding achievements, rice production still reveals many limitation and shortcomings. The growth in rice production over the past decades is mainly due to development in width whereas lack of attaching special importance to the depth, thus, rice planting uses huge resources with less economic efficiency and has negative impacts on the environment (ADB 2012, p.25). The production is still heavily based on empirical approaches, the income of rice grower is not remarkably improved leads to a movement of labors from agricultural to other industries. The problem of land management is not strict and clear enough as well as the industrialization and urbanization progress bring the challenges of reducing rice-planted area in the following years (Kompas *et al.* 2012, p.488).

To cope with these limitation and shortcomings, the need of maximizing productivity and capacity utilization in demand for inputs in rice production is considered one of primary issues for Vietnam agricultural development policy. The main concerns are: (i) What are the appropriate forms of CES function for rice production in Vietnam? (ii) What is the value of the elasticity of substitution of these CES models? (iii) Which policy implications could be deduced from this regard to the case of Vietnam? (iv) Are there any further issues during estimating the CES function of rice production in Vietnam?

The aim of this study is to address these questions by first identifying which CES models are appropriate for Vietnam s' rice production. Then, the study estimates the elasticity of substitution of these CES models. To be able to identify the CES models, it is necessary to test possible nested structures of three inputs: capital, labor and land. The method approach adopted here is to employ the nonlinear least square approach through the Levenberg-Marquardt method.

Estimating Vietnam's CES production function has two main motivations. Firstly, analyzing Vietnam s' rice production function provides the assessment of the substitutability between inputs, then, to find out the capacity utilization in the demand for those inputs. Secondly, an estimate of constant elasticity of substitution for Vietnam rice production is necessary to partly contribute to Vietnam's policy makers in designing appropriate policies in rice production to improve its effectiveness towards a sustainable development.

Despite the importance of estimating the CES models in rice production, there are as yet no studies on this topic. In fact, most models applied in the econometric analysis, particularly in macroeconomics topic and growth theory in which the Cobb-Douglas function is replaced with the CES production function. For example, Papageorgiou and Sam (2008) analyze the two-level CES production in the Solow models as well as diamond growth models. Caselli and Coleman (2006) assess the cross-country relationship between aggregate inputs and aggregate output through CES production function. In micro-macro model, CES is popularly applied to examine the linkage and interaction between the microeconomics model of firms and consumers with the macroeconomic model. Klump R (2007) is a great example of this. The paper examines the growth rate pattern of technical progress, labor and capital in order to estimate the supply-side system from the US economy in the period 1953-1998.

For the case of Vietnam's rice production, this paper makes the first attempt to estimate its CES production function with the objective of finding out the elasticity of substitution for that. This paper employed the data from the Vietnam Household Living Standard Survey (VHLSS) in 2012. The nonlinear least square method with Levenberg-Marquardt approach was applied to estimate the CES function. The rice production output is used as dependent variable and the variables: capital, labor and land are used as explanatory variables. This study adds to the literature by a construction of a Labor index for Vietnamese rice production function. Furthermore, it examines the nested CES model to assess which nested structures are appropriate for Vietnam's rice production. In addition, it performs the grid search method to re-assess the results of CES estimation and also alleviate some limitations of non-linear least square method.

The key finding of the study is that the estimated value of the elasticity of substitution lies between 0.44 and 0.46 which indicates the weak substitutability between land and the nest (labor, capital). Moreover, the study also finds out that the nested structure in which capital with land are nested and labor plays a role as the third input is rejected. This suggests that in the case of Vietnam's nested CES production function, it is impossible to take labor as the substitutable factor for land and capital.

The remainder of the paper is structured as follows. Section 2 describes the data collection and outlines the variable construction process as well as the model specification to estimate the elasticity of substitution of rice production in Vietnam in 2012. The results are presented in section 3. Section 4 discusses the results and concludes.

2 Method

2.1. Model specification

2.1.1 Nested CES models

The Cobb-Douglas function is the most well known functional form of production function in economic by Cobb and Douglas (1928) that is used to express the technological correlation between inputs and output. The inputs here are often capital and labor. However, this form is employed under the strong presumption of an elasticity of substitution which must be equal one, in other words, this production function has constant returns to scale (Cobb and Douglas 1928, p.141). This means when we double the consumption of capital and labor, then it will also double output. The CES function generalizes the Cobb-Douglas function and accepts any positive elasticity of substitution. It was first introduced by Solow (1956) and then was developed by Arrow *et al.* (1961). Its 2-input formula is specified as:

$$Q = \gamma[\delta \, x_1{}^{-\rho} + (1-\delta] \, x_2{}^{-\rho}]^{-\frac{v}{\rho}} \quad (1)$$

Where Q is the output quantity; x_1, x_2: input quantities; γ, δ, ρ: are parameters; γ indicates the productivity ($\gamma \epsilon [0, \infty)$); $\delta \epsilon [0, \infty)$ indicates the inputs' optimal distribution; ρ ranges between

$[-1,0)\cup(0,\infty)$ and ρ will define the elasticity of substitution $\sigma = \frac{1}{1+\rho}$; parameter $\upsilon \in [0,\infty)$ is the elasticity of scale. One point should be noted here is that the CES production function introduced by Arrow et al (1961) only considers the constant return to scale, thus, to allow for increasing or decreasing return to scale Kmenta (1967) put the parameter υ. The CES function will has increasing returns to scale if $\upsilon > 1$ and decreasing returns to scale if $\upsilon < 1$.

For multiple inputs, the general formula is identified as:

$$y = \gamma\left(\sum_{i=1}^{n} \delta_i x_i\right)^{-\frac{\vartheta}{\rho}} \tag{2}$$

Where, n is number of input factors, x_i is the quantity of input i.

However, this basic CES production function has limitation of defaulting equal substitution elasticity between input factors. Thus, Sato (1967) introduced the nested CES functions to remove this limitation. The general idea of the nested models is to group the inputs with same substitution and combine them with other input groups. In other words, nested models aim to create more levels of CES which are divided as upper level and lower level. The inputs in upper level group in CES function could be replaced with other inputs in lower level group. The nested CES model now becomes popular when we target to examine the input factors that might have further differentiation (Sato 1967, p.210)

The three-input nested CES function could be specified as:

$$y = \gamma[\delta(\delta_1 x_1{}^{-\rho_1} + (1-\delta_1)x_2{}^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)x_3{}^{-\rho}]^{-\upsilon/\rho} \tag{3}$$

Where y is output; x_1, x_2, x_3 are inputs; $\gamma, \delta, , \delta_2\ \rho, \rho_1$ are parameters; γ indicates the productivity ($\gamma \in [0,\infty)$); $\delta, \delta_1 \in [0,1]$ indicates the inputs' optimal distribution; ρ, ρ_1 ranges between $[-1,0)\cup(0,\infty)$ and ρ will define the elasticity of substitution = $\frac{1}{1+\rho}$; υ indicates the elasticity of scale.

2.1.2 Nested CES models of Vietnam's rice production

In this section, we examine a two level production function with three inputs: capital (KA), labor (LB) and land (LA). The first level function of nested CES function with input KA and LB is specified as:

$$M = (\delta_1 KA^{-\rho_1} + (1-\delta_1)LB^{-\rho_1})^{-\frac{1}{\rho_1}} \qquad (4)$$

This first level function will be nested with input variable LA to form the second level of CES function:

$$QX = \gamma[\delta M^{-\rho} + (1-\delta)LA^{-\rho}]^{-\upsilon/\rho} \qquad (5)$$

Putting equation (4) into (5) yields the two-level nested CES function with three inputs capital, labor and land. We denote this structure as (KA,LB)LA:

$$QX = \gamma[\delta(\delta_1 KA^{-\rho_1} + (1-\delta_1)LB^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)LA^{-\rho}]^{-\upsilon/\rho} \qquad (6)$$

Where, QX is the output quantity; KA, LB, LA are input quantities of capital, labor and land respectively; $\gamma, \delta, \delta_1, \rho, \rho_1$: are parameters; γ indicates the productivity ($\gamma \epsilon [0,\infty)$); $\delta, \delta_1 \epsilon [0,1]$ indicates the inputs' optimal distribution; ρ, ρ_1 ranges between $[-1,0) \cup (0,\infty)$ and ρ will define the elasticity of substitution $\sigma = \frac{1}{1+\rho}$

Similarly, the other two nested structures (LA,LB)KA and (KA,LA)LB are respectively identified as:

$$QX = \gamma[\delta(\delta_1 LA^{-\rho_1} + (1-\delta_1)LB^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)KA^{-\rho}]^{-\upsilon/\rho} \qquad (7)$$

$$QX = \gamma[\delta(\delta_1 KA^{-\rho_1} + (1-\delta_1)LA^{-\rho_1})^{\frac{\rho}{\rho_1}} + (1-\delta)LB^{-\rho}]^{-\upsilon/\rho} \qquad (8)$$

The study examines all three nested CES functions for Vietnam's rice production to estimate the elasticity of substitution of nested CES function $\sigma = \frac{1}{1+\rho}$. Specifically, it estimates two kinds of the elasticity of substitution: Hicks-McFadden elasticity of substitution and Allen-Uzawa elasticity of substitution. According to McFadden (2011), Hicks-McFadden elasticity of substitution is the direct elasticity of substitution which measures the elasticity of substitution only between the nested inputs. For example, in the nested model as (KA,LB)LA in equation (6), it measures the elasticity of substitution between capital and labor and it is specified as:

$$\sigma_{KA,LB} = \frac{1}{1 + \rho_1}$$

Allen-Uzawa elasticity of substitution, on the other side, evaluates the partial elasticity of substitution which measures the elasticity of substitution between the nested inputs and the third input (Uzawa 1962, p.295). For the nested model (KA,LB)LA, it measures the elasticity of substitution between the nest (capital, labor) and land. Allen-Uzawa elasticity of substitution is defined as:

$$\sigma_{KA\ LB,LA} = \frac{1}{1 + \rho}$$

2.2 Data

This study uses the latest full data set from Vietnam Household Living Standard Survey in the year 2012. This survey was conducted by Vietnam General Statistics Office with the support of the World Bank to collect the information on living standards of many households over provinces and cities of the country based on detailed questionnaires. The original data set included more than 27,000 households being interviewed. By filtering data, a number of households with not available data were eliminated, thus, reducing the size of data to be used in this study to more than 14,000 households. The available data for constructing the capital variable is taken from each household's cost of rice production. Thus, the capital variable is the sum of following costs: Cost of Seedling, sapling, small equipments, chemical fertilizer, insecticide, herbicide chemicals, electricity, gasoline, fee of repairing, maintenance and other costs. The available data of lands includes lands used for ordinary rice and glutinous rice production. The available data for output variable comprises of outputs of ordinary rice and glutinous rice production. The available data for labor variable includes the amount of money that each household used for hiring additional labors and the number of working days in rice production of each member in one household. They are used in constructing the Labor index which is described in detail in section 2.3.

Table 1 **Lists of variables used in the model**

Variable	Short	Definition	Source	Unit
Output	QX	Gross quantities of rice production	VHLSS files	kilograms
Capital	KA	Total fixed capital stock	VHLSS files	Thousands VND
Labor	LB	Labor index	VHLSS files	Unit of labor
Land	LA	Total lands used	VHLSS files	Square meters

2.3 Construction of Labor index for Vietnamese rice production function

The Vietnam Household Living Standard Survey (VHLSS) 2012 provides the information on labor participated in rice production including: (1) The amount of money that one household used for hiring additional labors (2) The number of working days in rice production of each member in one household (GSO 2012). Basing on this information, the study constructs a Labor index for Vietnamese rice production function in the year 2012 as follows:

Step 1: Summing up the working days of the all people from each household spent on rice production, we obtained WORKDAY_i where i denotes each household

Step 2: Summing up all working days of all households on rice production, we obtained the total time consumption on rice production from all households' members: $\sum_i^N \text{WORKDAY}_i$ where i denotes each household and N denotes total number of households.

Step 3: Dividing the total working days of all households by the number of households, we obtained the average time spending on rice production. We set it as working hour's standard for a standard labor or one unit of labor index. In this paper, a standard labor in rice production is assumed as the person who spends 173.67 days on rice production.

$$\overline{WORKDAY} = \frac{\sum_i^n \text{WORKDAY}_i}{N} = 173.67 \text{ days}$$

Step 4: Dividing the total working days of each household spending on rice production by the average time consuming on rice production to have the $Labourindex1$

$$Laborindex1_i = \frac{\text{WORKDAY}_i}{\overline{WORKDAY}}$$

Step 5: Beside each household's members participated in rice production, many households hired additional labors outside to supplement their production. The $Laborindex2$ calculates the hired outside labors for rice production of each household. The available data in VHLSS files contains the annual rent of outside labors of each household:$RENTLABOR_i$

First, we convert the rent of outside labors into feasible working days that outsides labor will supplement to each household's rice production. To do this, we must calculate the daily average income that the normal farmers could have. According to GSO (2012), the average monthly earnings of worker in agricultural sectors in the year 2012 is 2,543,000 VND. Thus, the standard daily income that the normal farmers in rice production should equal $(2{,}543{,}000\ \text{VND x12months})/\ 365\text{days}\ =\ 83{,}605.48\ VND$

Next, dividing annual rent of outside labors of each household by $83{,}605.48\ VND$, we obtain the $Laborindex2$ or the working days that the additional labors supplemented to the households' rice production.

$$Laborindex2_i = RENTLABOR_i / 83{,}605.48$$

Step 6: Finally, the $Laborindex$ of houshouse i is calculated as the sum of the $Laborindex1$ and $Laborindex2$ of this household.

$$Laborindex_i = Laborindex1_i + Laborindex2_i$$

2.4 Levenberg-Marquardt method

Among the methods for the estimation of constant elasticity of substitution, the maximum likelihood estimation, linear Taylor-series approximation and nonlinear least squares are the most popular. Compared to two other methods, nonlinear least square approach is considered more effective, especially for the case of having more than two independents variables in CES (Hoff 2004, p.297).

In linear Taylor-series approximation, the Kmenta approximation is the best representative (Heninningsen and Heniningsen 2012, p12). Kmenta (1967) derived the linear approximation of two-input CES production function by logarithmizing the CES function and then using the second-order Taylor series expansion. The advantage of Kmenta approximation is that from the linear approximation we could estimate the CES production function by ordinary least-squares techniques. This is also efficient to test whether the CES production function is under Cobb-Douglas form or not. However, applying the approximation methods to linearise the CES function exposes many problems. Kmenta (1967) himself confirms in the case that the elasticity of substitution is at the extreme (very low or very high), then his estimation might not produce reliable results. Other also proves this approximation is unsuitable when examining the CES production is not under the form of Cobb-Douglas function (Maddala and Kadane 1967, Thursby and Lovell 1978, Heninningsen and Heniningsen 2012). To overcome these problems of Kmenta approximation approach, many researchers have tried to estimate the linear system of equations which derived from cost minimization approach. However, this estimation often needs comprehensive price data which is, in most cases, usually difficult to get and might create the additional measurement error.

The nonlinear least square approach exposes numerous advantages when addressing the estimation of CES production function with n-inputs (Heninningsen and Heniningsen 2012, p.23). Also, in the nonlinear least square approach, The Levenberg-Marquardt method is considered the most effective method for estimating the CES function (Hoff 2004, p.299). The paper will employ the Levenberg-Marquardt curve-fitting method as the main approach to the find out the estimation of CES parameters. The Levenberg-Marquardt curve-fitting method fundamentally is the connection of two method s' approaches: The Gradient Descent method and the Gauss-Newton method (Marquardt, D 1963, p.437). This method procedure is described as follows:

Assume a non-linear function (CES function) to be estimated has the form: $\hat{y}(x_i; b)$ for the function $y(x_i)$ where x_i are independent variables and b is parameters vector. The target is that we should minimize the sum of weighted residuals between the data points and curve-fit functions, in other words, to perform least squares estimation.

Consider the chi-squared error criterion:

$$\chi^2(b) = \sum_{i=1}^{m} \left[\frac{y(x_i) - \hat{y}(x_i;b)}{w_i}\right]^2$$

$$= \left(y - \hat{y}(b)\right)^T W(y - \hat{y}(b))$$

$$= y^T W y - 2y^T W \hat{y} + \hat{y}^T W \hat{y} \tag{1}$$

Where $w_i$is error measurement of $y(x_i)$, W is the weighted matrix in which W_{ii} equals $\frac{1}{w_i^2}$. This criterion aims to seek the perturbation $\boldsymbol{\theta}$ for the parameters b which will minimize$\chi^2(b)$.

The Gradient Decent Approach: The purpose of this method is to force the updating parameters in the opposite sign of gradient from the chi-square objective function.

$$\frac{\partial}{\partial b}\chi^2 = \left(y - \hat{y}(b)\right)^T W \frac{\partial}{\partial b}\left(y - \hat{y}(b)\right)$$

$$= -\left(y - \hat{y}(b)\right)^T W \left(\frac{\partial \hat{y}(b)}{\partial b}\right)$$

$$= -\left(y - \hat{y}(b)\right)^T W\boldsymbol{J} \tag{2}$$

In which $\boldsymbol{J} = \frac{\partial \hat{y}(b)}{\partial b}$ is the Jacobian matrix that reflects the marginal changes in $\hat{y}(b)$to the variation of parameters b. From this, the perturbation $\boldsymbol{\theta} = \alpha \boldsymbol{J}^{\boldsymbol{T}} W(\boldsymbol{y} - \hat{y})$ will update the parameter b toward the steepest decent. Scalar α here is the determination of steps in the steepest-descent control.

The Gauss-Newton Approach: This method aims at minimizing sum-of-squares of a function which is assumed to be nearly quadratic with parameters at the points near the optimal bound. We have the estimated function in which the parameters is perturbed is $\hat{y}(b + \theta)$**.** From the first-order Taylor approximation series expansion, it is specified as:

$$\hat{y}(b + \theta) \approx \hat{y}(b) + \left[\frac{\partial \hat{y}}{\partial b}\right]\theta = \hat{y} + J\theta \tag{3}$$

Replace $\hat{y}(b)$ with $\hat{y}(b + \theta)$ in the equation (1):

$$\chi^2(b + \theta) \approx y^T W y + \hat{y}^T W \hat{y} - 2y^T W \hat{y} - 2(y - \hat{y})^T W J\theta + \theta^T J^T W J\theta \tag{4}$$

This equation indicates χ^2 is nearly quadratic in the θ and the Hessian in this case is approximate $J^T WJ$. Then the estimated θ which minimize $\chi^2(p + \theta)$ is identified as:

$$\frac{\partial}{\partial b}\chi^2(b + \theta) \approx -2(y - \hat{y})^T WJ + 2\theta^T J^T WJ = 0 \tag{5}$$

Or

$$[J^T WJ]\theta = J^T W(y - \hat{y})^T \tag{6}$$

Equation (6) will give us the estimated the Gauss-Newton perturbation.

The Levenberg-Marquardt Method will combine both the Gradient Decent approach and the Gauss-Newton approach, therefore, it lets the parameters moving in the range that updated from both gradient descent method and Gauss-Newton method. Specifically, we have:

$$[J^T WJ + \lambda I]\theta_{LM} = J^T W(y - \hat{y})$$

Where, λ is the algorithmic parameter. From this, we have Levenberg-Marquardt solution for non-linear least squares by the relationship:

$$[J^T WJ + \lambda diag(J^T WJ)]\theta_{LM} = J^T W(y - \hat{y})$$

2.5 Grid search

Estimating CES function through non-linear least square method could lead to the problem of flattening the surface of area around the minimal value of residuals when using the wide range of substitution parameters ρ, ρ_0, ρ_1. Grid search is the advanced and efficient method to alleviate this problem. The process of grid search is following: Firstly, the grid of values for ρ, ρ_0, ρ_1 is pre-selected which expectedly result in smallest sum of square of residuals. Then, holding those pre-selected values of substitution parameters fixed, we estimate the remaining parameters by using non-least square (Henningen and Heningen 2011, p.30). The purpose of grid search is try to find the best value of remaining parameters which result in smallest sum of square of residuals. The idea of this method is that the pre-selected grid values of substitution parameters will actively limit the surface of area around minimum, through this, when the non-linear least square operates again, the remaining parameters will be consequently produced on the ground of this limited surface around minimum.

3 Results

3.1 Descriptive statistics

The summary statistics for Vietnam s' rice production quantity (kilograms), weighted index for labor, the area of land for rice production (meters) in the year 2012 are reported in the table 1.

Table 1 **Summary statistics for Vietnam's rice production**, 2012

Variable	Mean	Standard deviation	Minimum	Maximum
Output quantity (QX), kilograms	4729.88	9943.24	65	333450
Labour (LB), Labour index	1.28	1.38	0.00172	14.79
Capital (KA), '000 VND	13491.59	29678.9	109	924098
Land (LA), square meters	8731.93	15938.28	120	507000

In 2012, on average annually one Vietnamese household produced around 4,729 kilograms of rice, the highest output quantity is 333,450 kilograms and the lowest is only 65 kilograms. The labor index has the highest value at 14.79 and the lowest value at 0.00172. The average labor index is 1.28 which means that on average each household has over one standard labor working in rice production. A standard labor or one unit of labor index in rice production is assumed in our model is the person spent on average 173.67 days per annual on rice production as explained in section 2.3. On average annually each household used 8,731 square meters of lands for rice production and spent 13,491 thousand VND on capital for rice production. To estimate Vietnam's nested CES production function, the rice production output is used as dependent variable and the variables: capital, labor and land are used as explanatory variables.

3.2 Estimating Vietnam s' nested CES rice production function

In this study, the statistical software R project (version 3.2.2) was used to perform the CES estimation and grid search process. Specifically, in R project, we use the R-package micEconCES created by Henningen and Heningen (2011). The R-package micEconCES is mainly designed to estimate the CES production function and currently this is one of the most the updated and efficient programs for estimating the CES (Shen and Whalley 2013, p.7) The estimation results are represented as follows: First, the general results of estimating CES production function with different nested structure will be checked to find out the suitable nested structures. Then, the detailed results for each suitable nested structure will be examined. Second, the grid search is preformed to provide more accurate results of CES estimation. One point should be noted that all estimation processes, grid search are performed by the Levenberg-Marquardt method programmed under R-package micEconCES.

3.2.1 Choosing suitable nested CES structure for Vietnam s' rice production function

The task of this part is to find out which nested CES structures are suitable for Vietnam s' rice production function. Table 2 compares the estimates results of CES function with different nested structures in both cases of constant return to scale and variable return to scale. The CES function with constant return to scale has the assumption that the parameter υ is unity whereas the CES function with constant return to scale has no condition on υ. The results shows that in both cases of the constant return to scale and the variable return to scale, the nested structure (Ka,Lb)La and (La,Lb)Ka are suitable while the structure (Ka,La)Lb is not reliable and reflects less economic meaning for Vietnam s' CES rice production function.

Table 2 **Estimating Vietnam s' CES rice production function with different nested structure**

	Constant return to scale (with $\upsilon = 1$)				Variable return to scale (without $\upsilon = 1$)			
Nest Structure	Estimate of $\sigma_{12,3}$	Standard error of $\sigma_{12,3}$	R^2	Residual standard error	Estimate of $\sigma_{12,3}$	Standard error of $\sigma_{12,3}$	R^2	Residual standard error
(Ka,Lb)La	0.443	0.0154	0.975	1554.22	0.445	0.0151	0.977	1508.80
(La,Lb)Ka	0.4401	0.0132	0.975	1554.73	0.445	0.0125	0.977	1508.368
(Ka,La)Lb	2.829	84.245	0.975	1554.658	0.880	0.0280	0.369	7899.023

Note: Only $\sigma_{12,3}$ Allen-Uzawa (partial) elasticity of substitution is checked
$\sigma_{12,3}$ is the elasticity of substitution in which input 1 and input 2 are nested but input 3 is not.

In the case of constant return to scale, the results prove that the nested structure (Ka,La)Lb does not suit to the case of nested CES function while the nested structures (Ka,Lb)La and (La,Lb)Ka are the suitable models. All three nest structures have high value of R^2 which is rounded to 0.975. The R^2 value of 0.975 means that approximately 97.5 per cent of variations in the Vietnam s' rice production output could be explained by the variables: capital, labor and land. Thus, the higher value of R^2, the better the models fit the data (Heij *et al.* 2004, p.210). However, in the nest structure (Ka,La)Lb the estimate of the Allen-Uzawa (partial) elasticity of substitution $\sigma_{12,3}$ is considerably high which is around 2.829. Normally, to have economic meaning, $\sigma_{12,3}$ should lie in the range [0,1] (Henningen and Heningen 2011, p.16). Such high value of $\sigma_{12,3}$ has no application to reality. Moreover, standard error of $\sigma_{12,3}$ in the nest (Ka,La)Lb is estimated approximately 84.245 which is much higher than ones in other two nested structures. Theoretically, high standard error usually implies that the estimates might have high variances or might be resulted from the small sample size (Heij et al. 2004, p.120). In this study, the data is quite large and sufficient enough, thus, the high variances of estimates would be the main problem. Therefore these evidence indicates that the nested structure (Ka,La)Lb does not suit to the case of the nested CES function with constant return to scale. Also, they suggests the CES production function with the nested structures (Ka,Lb)La and (La,Lb)Ka are the suitable models for the case of Vietnam's rice production function in 2012.

The nested CES production function with variable return to scale has the same conclusion with the case of constant return to scale. The nested structure (Ka,La)Lb is inappropriate for Vietnam's nested CES rice production function while the nested structures (Ka,Lb)La and (La,Lb)Ka are proved to be the suitable ones. From the table 2, both the nested structures (Ka,Lb)La and (La,Lb)Ka have the same goodness of fit (R^2) which is at high value of 0.977. However, the nested structure (Ka,La)Lb has low R^2 which is approximately 0.369. Such low value of R^2 means only 36.9 percent of variations in the Vietnam s' rice production output could be explained by the variables: capital, labor and land. This poor result indicates that estimation of the nest structure (Ka,La)Lb is unreliable and this structure will not be chosen for estimating nested CES function.

For evidences stated above, the study chooses the nested structures (Ka,Lb)La and (La,Lb)Ka for further assessment to estimate Vietnam's nested CES production function. The nested structure (Ka,La)Lb is not suitable due to its unreliable and meaningless results. The next section will focus on examining in detail the estimates of two nested structures (Ka,Lb)La and (La,Lb)Ka for both cases of constant return to scale and variable return to scale.

3.2.2 Estimating Vietnam s' nested CES rice production function with constant return to scale

After choosing the suitable nested structures are (Ka,Lb)La and (La,Lb)Ka , the next step is to examine their estimation results in detail.

Table 3 **Estimate nested CES function with constant return to scale**

	Estimates		Standard Error		T value		Pr (>|t|)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.377	0.453	1.361	16.221	0.277	0.028	0.782	0.978
delta_1(δ_1)	-0.281	-0.145	4.444	64.886	-0.063	-0.002	0.950	0.998
Delta (δ)	0.613	0.597	1.748	18.351	0.351	0.033	0.726	0.974
rho_1(ρ_1)	-0.468	-0.824	1.377	44.77	-0.340	-0.018	0.734	0.985
Rho (ρ)	1.255	1.272	0.0782	0.0679	16.050	18.726	<2e-16 ***	<2e-16 ***
$\sigma_{1,2}$	1.879	5.683						
$\sigma_{12,3}$	0.443	0.44						

	Nested CES structures	
	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1554.22	1554.73
R^2	0.975	0.975

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution
$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution
***indicates significance at 1 percent level

There is not much difference in estimates of CES production function with nested structure (Ka,Lb)La and (La,Lb)Ka. They both have the Allen-Uzawa (partial) elasticity of substitution around 0.44. Two nested structure have the same goodness of fit (R^2) and residual standard errors. The R^2 value of 0.975 means that approximately 97.5 per cent of variations in the Vietnam s' rice production output could be explained by the variable capital, labor and land. This proves that the estimates are relatively reliable and the models fit well with the data. The Allen-Uzawa (partial) elasticity of substitution is smaller than one which implies the weak substitutability between land and the nest (capital and labor).

3.2.3 Estimating Vietnam s' nested CES rice production function with variable return to scale

Table 4 **Estimate nested CES rice production function with variable return to scale**

	Estimates		Standard Error		T value		Pr (>|t|)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.5018	0.618	0.30	0.258	1.671	2.393	0.0948	0.0167 *
delta_1(δ_1)	0.91	0.627	2.224	0.812	0.407	0.772	0.684	0.439
Delta (δ)	0.04	0.149	0.685	0.431	0.05	0.347	0.953	0.728
rho_1(ρ_1)	-1.055	-0.575	3.856	0.366	-0.274	-1.572	0.784	0.116
Rho (ρ)	1.25	1.248	0.0762	0.0634	16.340	19.677	<2e-16 ***	<2e-16 ***
Nu (ν)	1.036	1.0376	0.00126	0.00136	821.497	763.918	<2e-16 ***	<2e-16 ***
$\sigma_{1,2}$	NA	NA						
$\sigma_{12,3}$	0.445	0.445						

	Nested CES structures	
	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1508.80	1508.368
R^2	0.977	0.977

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution
$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution
***and * indicates significance at 1 percent level and 5 per cent level respectively

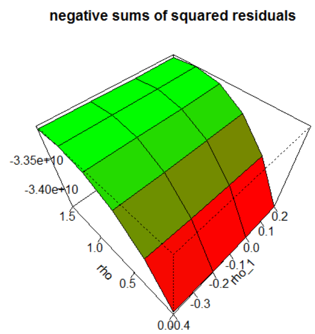
The Allen-Uzawa (partial) elasticity of substitution is estimated around 0.445 for both nested structures (Ka,Lb)La and (La,Lb)Ka. Again, it indicates that land and the nest (capital and labor) seems to be difficult to substitute with each other. The estimates are relatively reliable with high value of R^2. Land, labor and capital together explain around 97.7 per cent of the variation in the rice output quantities.

3.3 Grid search for Vietnam s' CES rice production function

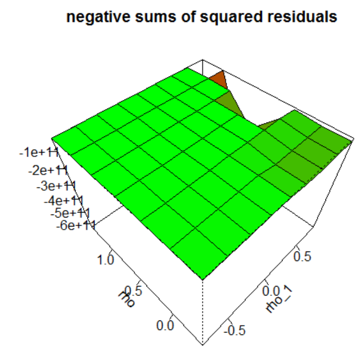
This section examines the nested CES production function again with the attempt to see how estimated results change when we perform the grid search method. Usually, one problem that estimating CES production often has is that if we allow substitution parameters ρ, ρ_0, ρ_1run in the wide ranges of number, this consequently tends to flatten the optimal area of residuals. Thus, this might distort the estimates. Grid search method alleviates this problem through narrowing down the range of substitution parameters and using this range to re-estimate CES function.

3.3.1 Choosing pre-selected grid of values for substitution parameters

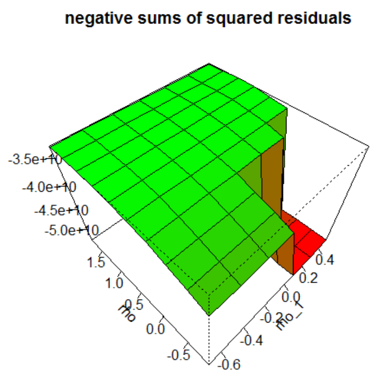
Figure 1: **Grid search for values of substitution parameters**



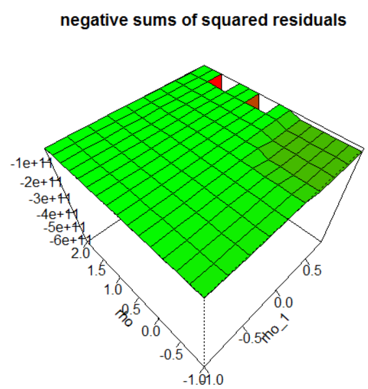
(a)



(b)



(c)



(d)

Source: Author s' estimates

Note: Graph (a): rho1 (ρ_1) (from-0.4 to 0.3, with an increment 0.2); rho (ρ) (from 0 to 1.5, with an increment 0.3)
Graph (b): rho1 (ρ_1) (from -0.8 to 0.9, with an increment 0.2); rho (ρ) (from -0.4 to 1.5, with an increment 0.3)
Graph (c): rho1 (ρ_1) (from -0.7 to 0.6, with an increment 0.2); rho (ρ) (from -0.9 to 2, with an increment 0.3)
Graph (d): rho1 (ρ_1) (from -1 to 0.8, with an increment 0.2); rho (ρ) (from -1 to 2, with an increment 0.2)

There is no software or program designed to automatically find out the pre-selected values for substitution which will be used as inputs for the grid search method. The proper pre-selected values for substitution are usually chosen basing on the graphs of grid search and compare the results (Henningen and Heningen 2011, p.38) Figure 1 describes the different combination of negative sum of squares and arbitrary grid values of substitution parameters (ρ, ρ_1). Each graph is constructed by three axes. The vertical axis measures the negative sum of square residuals and the other two measure the value of ρ_1 and ρ .The ranges of grid values of substitution parameters is selected randomly and also guessed basing on the normal CES estimates. While graphs (b), (c), (d) reflects the "flatter" surface around the maximum of negative sum of square, the graph (a) expresses the optimal area of the negative sum of square. The larger negative sum of square means the smaller absolute value of the sum of square residuals. Thus, the graph (a) with a concave shape shows that the sum of square is smallest and achieves optimal value when the value of ρ_1 ranges from-0.4 to 0.3 with an increment of 0.2 and the value ρ from 0 to 1.5 with an increment 0.3. Hence, the range values of (ρ, ρ_1) from graph (a) is the most proper pre-selected values for grid search to find out the best value of the elasticity of substitution.

3.3.2 Re-estimate nested CES function with grid search method

After selecting the pre-selected grid values of substitution parameters, the next step is to re-estimate CES function using these pre-selected values to produce more accurate value of σ. The pre-selected range of values that the value of ρ_1 ranges from-0.4 to 0.3 with an increment of 0.2 and the value of ρ_0 from 0 to 1.5 with an increment 0.3 will be chosen in this section. The re-estimated results from table 5 and table 6 show that the value of the Allen-Uzawa (partial) elasticity of substitution lies between 0.45 to 0.53. This again confirmed the weak substitution of land and the nest (labor, capital). The high value of R^2 which is around 0.98 proves the best fit of estimated models. These estimates re-affirm the accuracy of previous results before using grid search. Therefore, it helps to see that in this case, the problem of flattening the optimal area of residuals which the CES estimation might not much affect the estimates.

Table 5 **Re-estimate nested CES function with constant return to scale**

	Estimates		Standard Error		T value		Pr (>$\|t\|$)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.158	1.02e-05	0.164	2.747e-03	0.965	0.004	0.335	0.99704
delta_1(δ_1)	-0.527	-9.19e+01	0.632	1.014e+04	-0.835	-0.009	0.404	0.99277
Delta (δ)	0.789	1.000e+00	0.188	3.403e-04	4.188	2938.858	2.82e-05 ***	<2e-16***
rho_1(ρ_1)	-0.20	-4.000e-01	0.0776	1.225e-1	-2.575	-3.265	0.010 *	0.00109 **
Rho (ρ)	0.90	1.200e+00	0.0642	6.687e-02	14.029	17.945	< 2e-16 ***	<2e-16***
$\sigma_{1,2}$	1.25	1.667	0.121	0.34	10.30	4.898	<2e-16 ***	9.68e-07***
$\sigma_{12,3}$	0.526	0.454	0.0178	0.0138	29.62	32.9	<2e-16 ***	<2e-16 ***

	Nested CES models	
	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1549.768	1549.689
R^2	0.976	0.976

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution

$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution

***and * indicates significance at 1 percent level and 5 per cent level respectively

Table 6 **Re-estimate nested CES function with constant return to scale**

	Estimates		Standard Error		T value		Pr (>$\|t\|$)	
	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka	(Ka,Lb)La	(La,Lb)Ka
Gamma (γ)	0.334	0.455	0.0113	0.355	29.485	1.284	<2e-16 ***	0.199
delta_1(δ_1)	-0.00125	0.788	0.0065	0.771	-0.192	1.021	0.848	0.307
Delta (δ)	0.412	0.427	0.0225	0.520	18.269	0.821	<2e-16 ***	0.412
rho_1(ρ_1)	0.20	-0.40	0.461	0.467	0.434	-0.857	0.664	0.391
Rho (ρ)	1.20	1.20	0.0698	0.0621	17.189	19.318	<2e-16 ***	<2e-16 ***
Nu (ν)	1.035	1.037	0.00133	0.00138	779.928	753.156	<2e-16 ***	<2e-16 ***
$\sigma_{1,2}$	0.833		0.320		2.603		0.00923	
$\sigma_{12,3}$	0.454	0.454	0.0144		31.514		< 2e-16 ***	

	Nested CES models	
	(Ka,Lb)La	(La,Lb)Ka
Residual standard error	1508.50	1508.55
R^2	0.977	0.977

Note: $\sigma_{1,2}$: The Hicks-McFadden elasticity of substitution

$\sigma_{12,3}$: The Allen-Uzawa elasticity of substitution

***and * indicates significance at 1 percent level and 5 per cent level respectively

4 Discussion and conclusion

4.1 Comparison of the results of elasticity of substitution with other empirical estimates

The objective of this study is to estimate the elasticity of substitution of the nested CES function for Vietnam's rice production in the year 2012. The model shows that the estimated value of the elasticity of substitution lies between 0.44 and 0.46 which is less than 1. This indicates that there is a weak substitutability between land and the nest (labor, capital). It is worth comparing the estimate of elasticity of substitution for Vietnam rice production function found in this paper with other estimates from previous empirical studies. Table 7 shows some empirical estimates of the elasticity of substitution of CES models from previous papers.

Table 7 **The results of elasticity of substitution from other empirical estimates**

No	Sector	σ_{KL}	σ_{KLE}		
		Tsurumi (1970)	Kemfert (1998)	Van de Werf (2008)	Okagawa and Ban (2008)
		NLS	NLS	CM	CM
1	**Agriculture, forestry and fishing**				**0.39**
2	Mining and quarrying		0.56		0.55
3	Total Manufacturing	1	0.78	0.40	0.40
4	**Manufacture of food products, beverages products**	**0.50**		**0.29**	**0.64**
5	Manufacture of textiles, wearing apparel, leather and related products	0.84			
6	Manufacture of wood and wood products	0.23			0.46
7	Manufacture of paper products; printing and reproduction of recorded media	0.56			
8	Manufacture of coke and refined petroleum products	0.90			
9	Manufacture of chemicals and chemical products	0.41			-0.07
10	Manufacture of basic metals and fabricated metal products, except machinery and equipment	0.63	0.4	0.65	0.64
11	Manufacture of electrical equipment	0.27			0.52
12	Manufacture of machinery and equipment n.e.c				0.53
13	Manufacture of transport equipment	0.71	0.35	0.17	0.52
14	Electricity, gas, steam and air conditioning supply, Water supply; sewerage, waste management and remediation activities				0.26
15	Construction			0.29	0.53
16	Information and communication				0.52
17	Financial and insurance activities				0.32
18	Other service activities				0.78

Note: All industrial classifications follow the International Standard Industrial Classification of All Economic Activities (ISIC) of the United Nations (2008).

NLS: Nonlinear least square method.

CF: Cost function minimization method.

σ_{KL}: the elasticity of substitution of CES models with two inputs: capital and labor.

σ_{KLE}: the elasticity of substitution of CES models with three inputs: capital, labor and energy.

Tsurumi (1970) examined the CES models for twelve Canadian manufacturing industries with two inputs: capital and labor during the period 1926 -1939 and 1946-1967. She found out that the elasticity of substitution for similar industries as foods and beverage was around 0.5. Okagawa and Ban (2008) estimated the elasticity of substitution for nested CES production function using cost minimization method with three inputs (capital, labor and energy) for 19 industries from 14 countries in the period 1995 to 2004, in which the elasticity of substitution for agricultural industries was estimated at 0.39. Therefore, the estimates of the elasticity of substitution for Vietnam‘s CES rice production function at around 0.44 to 0.46 in this paper is consistent with other estimates from previous empirical studies.

4.2 The weak substitutability in the Vietnam‘s nested CES rice production function

The study results show that the estimated value of the elasticity of substitution for Vietnam rice production lies between 0.44 and 0.46 which indicates the weak substitutability between land and the nest (labor, capital). This result suggests that the chance to use additional land as the substitutable factor for the shortage of labor and capital is very low. In other words, it is difficult to compensate the shortage of labor and capital only by expanding the area of lands. Moreover, compared with other industries such as textile sectors and real estate sectors, the elasticity of substitution for rice production is lower. This implies that in rice production, it is relatively difficult to have substitutability between its input factors compared to other sectors. Further, the study also finds out that the nested structure in which capital with land are nested inputs while labor plays a role as the third input is rejected. This finding suggests it is impossible to take labor as the substitutable factor for land and capital.

These findings provide useful policy implications. An estimate of constant elasticity of substitution for Vietnam rice production is necessary to address the existing weakness of rice production in Vietnam and partly contribute to provide the useful empirical evidences for designing appropriate policies for rice production. First, the weak elasticity of substitution suggests that Vietnam rice production is not easy to have the substitutability between its inputs. Given that rice production is managed by the Vietnamese government, these findings will partly help policymakers to design the appropriate policies on rice production with the efficient proportion of inputs factors in order to get the optimal output. Second, the conclusion on rejected structure suggests that it is impossible to compensate the shortage in capita and land by only expanding the number of labors. The feasible policy might be considered is to increase the productivity through enhancing the technology applications. Third, the relatively weaker substitution of inputs in rice production compared with other industries will partly help

Vietnamese government to plan how to allocate inputs and resources between rice production with other industries. Finally, the estimates for Vietnam's CES production function could be taken as references in future research on the topic of Computable General Equilibrium (CGE) modeling.

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