Driving time, productivity, and the fundamental law of road congestion*

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Abstract

Road congestion is a negative externality associated to automobile use and can negatively affects utility in several directions: delay time, wasted fuel, but also can have a negative impact on labor productivity. This paper develops a Dynamic Stochastic General Equilibrium model to study the interactions between roads, traffic, congestion and productivity over the business cycle. Following a positive aggregate productity shock, traffic density and congestion tend to rise, so dampening its positive effects on aggregate activity. A recent article by Duranton and Turner (*American Economic Review*, 101, 2616-52, 2001) confirms empirically the so-called "fundamental law of highways congestion", which states that an increase in the stock of road produces a traffic density rise of same proportion, thus leaving congestion unaffected in the long run. We explore some fundamentals behind this fundamental law. We conclude that a prototype dynamic model may reproduce a rise in output in response to a road capacity expanssion and predicts a 1/3 rise in traffic, as a consequence of its positive impact on economic activity. Finally, we derive a Pigouvian tax schedule that internalizes the social costs of congestion. (*JEL* E32, R41, R42, R48)

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1 Introduction

Road congestion is a negative externality associated to automobile use and it can negatively affects welfare in several directions, including delay time and wasted fuel. According to a recent report by INRIX-CEBR (2015), people in Europe and the U.S. waste, on average, 111 hours annually in gridlock. As of 2013, traffic congestion costs for the U.S. are estimated to be 124,2 billion dollars. For European countries, estimated congestion costs are also large: 20.5 billion dollars for the U.K., 22.5 for France, and 33.4 for Germany. Just in Los Angeles, the estimate congestion cost is 23.2 billions dollars (INRIX-CEBR, 2015). In terms of annual hours wasted by passenger in 2013 were 68 in the U.S., 123,9 in the U.K., 118 in Germany and 135.8 in France. Duranton and Turner (2011) pointed out that in the year 2001, an average American household spent 161 person-minutes per day in a passenger vehicle. Similar conclusions are reported by Parry, Heine, Lis and Li (2014), who studied road traffic in several cities across the world. As of 1995, using the Milennium Cities Database, they estimate that the average delay time was 0.0058 hours per km. for 15 US cities. They also reported an average of 18.4 thousand kilometers driven. Multiplying these two figures, we obtain a total delayed hours per year of 107.3, a figure very similar to the estimated value in INRIX-CEBR (2015). Schrank and Lomax (2005) estimated that urban road congestion causes 3.7 billions hours of delay a year worldwide.

Congestion can be viewed as a function of traffic density relative to road capacity. When an additional vehicle incorporates in a road, from a certain level of traffic density, congestion increases as a higher proportion of road capacity is used, and thus, average speed decreases. When we add vehicles to a road, the distance between vehicles decreases and hence also decreases average speed, increasing travel time. If the number of vehicles in a particular road is low enough, the interactions among road users become smaller, resulting in a travel speed equals to the so-called "free flow vehicle speed" and no congestion.[1] However, as the number of drivers increases, interactions among them also increase, giving place to a negative externality in the form of higher driving time. Congestion costs have various consequences: time delayed, extra fuel consumption, higher vehicle operating costs and emissions and a likely increase of crashes.[2] Congestion costs heavily depend

[1] Free flow vehicle speed represents the speed at which drivers choose to travel without influence by other road users and their compliance with speed limits for a certain type of road.

[2] The relationship between congestion and fuel consumption is not clear. Several authors pay attention on the fact that congestion involves higher consumption of fuel, since fuel efficiency decreases. For instance, Schrank, Lomax and Eisele (2011) estimated that additional fuel consumption from congestion is only about 5% of total cost, accounting the remaining 95% for the cost of additional time lost. However, other authors, as Greenwood and Bennett (1996), and Small and Gómez-Ibáñez (1998), pointed out that the relationship between slower traffic speed and fuel consumption is difficult to specify correctly.

on the value of travel time, which is related to wages for each country.

Road congestion can be apparently solved by investing in new road capacity given traffic density. An early contribution, however, Downs (1962) conjectured that as road capacity increases, traffic also increases in the same proportion, so as to keep congestion constant in the long term. This phenomenon was called the "Fundamental Law of Road Congestion". This implies that building new roads does not help reduce congestion, as more traffic is generated in response to the new capacity. Jorgensen (1947) was the first to estimate empirically the response of traffic to road capacity expansions.[3] Following Litman (2016) generated traffic in response to road capacity expansion comes from two sources: Diverted traffic (trips shifted in time, route and destination, what Downs, 1992, call the *Triple Convergence*), and induced traffic (shifts from other modes, longer trips and new trips). Based on that Fundamental Law it results clear that generated traffic reduces benefits of road capacity expansion over congestion.

The fundamental law of traffic congestion raises an important policy recommendation: road congestion cannot be relieved by building new roads, whose construction is expensive. In this sense, the elasticity of traffic with respect to road enlargements is a central question in the debate about new road construction. A body of empirical literature has explored the relationship between road capacity, measured either by lane-kilometers or as travel time, and traffic, measured as vehicle-kilometer traveled (VKT). However, estimated generated traffic associated to road improvements are very imprecise and estimated elasticity goes from 0 to 100 percent. In general, the majority of empirical estimations only support partially the "fundamental law of highways congestion". Hansen (1995), for instance, reports values for the elasticity ranging from 0 to 0.3 one year after construction, and values from 0.2 to 0.8 four years later, using data from the California Metropolitan Areas. He finds, by contrast, that only a small fraction in the rise in VKT can be associated to the increase in lane-miles. Similarly, Cervero (2003), also using data from the California freeway, estimates a long-term elasticity of 0.64. Although estimated elasticity is always positive, they are usually below one.[4] Nevertheless, a recent article by Duranton and Turner (2011) estimated the elasticity of vehicle kilometers traveled with respect to lane kilometers, obtaining a value close to one for a number of alternative specifications. This is an empirical evidence in favor of the

[3] Jorgensen (1947) estimated the induced traffic from the construction of the Merritt and Wilbut Cross parkways in the corridor between New York city and New Haven, Connecticut. He estimated the relationship between traffic and gasoline sales in Connecticut before the construction of the parkways. Then, he used information on the growth in gasoline sales after the construction to estimate the growth in traffic, obtaining between 20 and 25 percent more traffic that would have been expected.

[4] See Cervero and Hansen (2002), and Cervero (2002) for surveys of the empirical literature, most of the works reporting elasticity estimates below one.

"fundamental law of highways congestion" as proposed by Downs (1962). Moreover, they suggest that the fundamental law can be extended to a broad class of major urban roads, resulting in a general "fundamental law of road congestion". Based on these results, Duranton and Turner (2011) conclude that road capacity expansion is not an appropriate policy to reduce traffic congestion, and that policy interventions aimed at pricing congestion, are instead the correct instrument to internalize social costs from congestion. These policy measures include tolls, entry restrictions, or Pigouvian taxation.

Road traffic and congestion problems have been studied in the literature using alternative models. Examples include those of Vickrey (1969), Henderson (1974, 1977, 1981), Newell (1987), Arnott, De Palma and Lindsey (1990, 1993, 1994), among others. These models focus on microeconomic aspects and fail in taking into account the behavior of the different agents in the economy at an aggregated level. This paper builds a dynamic stochastic general equilibrium (DSGE) model to study traffic and congestion over the business cycle. Automobile travelling activities are included in households' utility function in two ways. First, we assume that households receive services from driving cars. Second, total available time can be allocated into leisure, working activities and driving. Somewhat related to our model is the one by Wei (2013), who develops a DSGE model to study fuel efficiency. In our model, traffic is pro-cyclical as it is related to the economic activity given its complementarity with labor. This implies that congestion will be also pro-cyclical given the stock of lane kilometers. In fact, road congestion has decreased dramatically during the last downturn starting in 2008. On the other hand, different studies show that the stock of roads have increased at a lower rate than traffic has. For example, Parry, Walls and Harrington (2007) pointed out that between 1980 and 2003, urban vehicles miles traveled in the US increased by 111%, whereas road lane-mile capacity only increased by 51%, implying that annual urban congestion delays increased from 16 to 47 hours per driver. A result we find is that the effects of a positive aggregate shock are dampened as the level of congestion increases. Moreover, we find a negative effect on hours worked (and leisure) as additional time is wasted in driving. Therefore, congestion can be viewed not only as a negative externality related to road traffic affecting welfare but a factor slowing down business activity.

Our model helps to analyze the conditions under which the "fundamental law" is confirmed. One important result from our model is that increasing road stock also rises VKT and hence, generated traffic is endogenously produced. The rise in VKT is about 1/3 of the rise in the stock of road, and hence, the level of congestion is reduced as road investment is increased, a response consistent with most of the empirical literature. As drivers perceive the immediate benefits from extra reductions in delay time, traffic density increases due to a rise in the kilometers driven. In a later moment, this situation induces additional advantages to purchase new cars, so that the stock of cars increases, which further soars traffic density. There is an additional channel. Road is an input in the aggregate production function, which complements to capital and labor. Therefore an expansion in road capacity motivates economic activity. Fernald (1999) finds that vehicle-intensive industries benefit more from road-building. Public investment may account for a substantial share of the slowdown in productivity growth after 1973, when congestion became important. Fernald (1999) reports evidence that the road stock per capita remained almost constant during the period 1973-1994 while traffic grew steadily. Our recommendation differs from that of Duranton and Turner (2011), as our model hinges on these features that makes road investment a tool for boosting economic growth.

The model is thought to represent the congestion of urban areas. The central planner problem will be referred as the city mayor's problem: an agent that takes decision of consumption, road investment, hours worked, driving, while internalizing the social costs of congestion. This way, using the optimal allocations derived from the mayor's problem, we propose a Pigouvian tax schedule that internalizes congestion costs. We conclude that this can be done through a fuel tax.

The remaining of the paper is structured as follows. In Section 2 we describe the model. The calibration exercise is presented in Section 3. Section 4 studies the dynamic properties of the model in the short-run. Section 5 uses the model to theoretically substantiate the Fundamental Law of Road Congestion. Section 6 quantify the cost of congestion. Finally, Section 7 presents the main conclusions.

2 The model

In this section, we develop a dynamic stochastic general equilibrium (DSGE) model in which personal transport activities are included in household's utility function, road infrastructure in the production function, and road investment in the government (the city hall) budget constraint. Driving cars enters in the households utility function in two ways. First, households receives services from driving cars. Second, driving time is related to working time and labor is measured in efficient units where cars are considered as a working tools which complement labor hours. Public investment decisions are transformed in public capital (roads) stock. Finally, road infrastructure is considered as an additional input in the aggregate production function.

In the model described so far, lowercase letters will refer to individual variables, while uppercase letters refer to aggregate variables. This distinction is important given that individuals do not take into account the effects of their own driving decisions on aggregate traffic intensity.

2.1 Households

The representative household has a concave utility function which depends on consumption c_t, services from vehicles s_t, hours worked h_t,, and driving time d_t, respectively. The utility function is assumed to have the following functional form:

$$u(c_t, s_t, h_t) = \ln c_t + \varphi_s \frac{s_t^{1-\gamma} - 1}{1-\gamma} - \varphi_h \frac{(h_t + d_t)^{1+1/\upsilon}}{1+1/\upsilon}, \tag{1}$$

where $\gamma > 0$ denotes degree of concavity in the utility from vehicle services. The duet of parameters $(\varphi_s > 0, \varphi_h > 0)$ denote the willingness to drive and the willingness to work, respectively, and where υ is the Frisch elasticity of labor supply. Notice that working activities requires some additional non-paid time devoted to travel activities.[5]

Services from vehicles are assumed to depend on the flow kilometers driven, m_t, and on the current stock of cars, q_t:

$$s_t = q_t m_t^{\phi}, \tag{2}$$

where $0 < \phi < 1$, implying that using cars too intensely has diminishing returns i.e., it is better to use the fleet less intensively by having more cars.

Given that this paper is focused on congestion, driving time d_t will be limited on delay time due to highway congestion, so that leisure time while driving is excluded from this definition. Following Arnott, de Palma and Lindsey (1993), total driving time is the sum of free-speed driving time, which it is assumed to be fixed, plus congestion time, which depends on traffic density relative to road capacity. Notice that in our model, the value of travel time is defined endogenously. In the literature (mainly in Cost-Benefit Analyses), it is assumed that the opportunity cost of travel time is different from the opportunity cost of working time, although both are related to wages. Empirical literature uses a value of travel time half the market wage (see, for instance, Small and Verhoef, 2007). In general, the opportunity cost of working time is measured in lost wage. Here, the opportunity cost of travel time is defined in a somewhat different manner, as forgone wage and forgone leisure.[6] For the sake of simplicity, the economy's total available amount of discretionary

[5] Similar utility functions to (1) are those used by Parry and Small (2005) and by Wei (2012).

[6] In equilibrium, one unit of time of leisure is valued

time is normalized to one, which we assume to be distributed between leisure, working activities and driving time:

$$\text{leisure} = 1 - h_t - d_t. \tag{3}$$

Lets M_t and Q_t denote aggregate values of kilometers driven and the aggregate stock of vehicles, respectively, and let $K_{g,t}$ denote the stock of road available for drivers at time t. Traffic density is accounted by $Q_t M_t$. Road congestion is defined as traffic density relative to the stock of roads:

$$\text{congestion} = \frac{Q_t M_t}{K_{g,t}}.$$

In turn, we assume that time delay while driving exponentially depends on congestion:

$$d_t = \eta_0 \left(\frac{M_t Q_t}{K_{g,t}} \right)^{\eta_1}, \tag{4}$$

with $\eta_0 > 0$, and where $\eta_1 > 0$ represents the elasticity of driving time with respect to congestion. This specification for driving time is analogous to those used by transportation engineers for travel delay (Parry *et al.*, 2014). This way, delay time can be viewed as a negative externality from the congestion of public infrastructure. Under a decentralized framework, individuals' choices take road density $M_t Q_t$ as given, that is households do not consider their own use of car to affect traffic congestion.

In our model, driving activities correspond to commuter trips. Time spent commuting is not leisure, and therefore, implies a disutility. On the other hand, time spent driving is not productive (although compulsory) and hence, no direct income is produced by this allocation of non-leisure time. Furthermore, this activity implies additional costs, such as fuel consumption and other operational costs related to vehicles' use.

Following Fisher (2007), we allow a degree of complementarity between vehicles services, s_t, and the supply of labor, such that the amount of hours measured in efficiency units, $\widetilde{h}_t$, are given by:

$$\widetilde{h}_t = h_t^{\theta} s_t^{1-\theta}, \tag{5}$$

with $\theta \in [0, 1]$[7]. Such a complementarity between vehicle services and hours worked responds to

[7]Standard RBC models suppose $\theta = 1$, implying that effective hours worked equal hours devoted to non-leisure activities. However, this case makes consumption of durables decrease in response to a positive shock to TFP, a prediction not supported by the data. Assuming, instead, $\theta < 1$ helps the standard model to reconcile with the data (see Fischer, 2007).

the idea that cars can be viewed as work tools and are not mere consumption durable goods. In this respect, we follow Fisher (2007) and assume that the utilization of automobiles affects productivity and increases hours worked in efficiency units.

To render proper services, cars need to use resources that affect household's budget. We assume that these expenditures can be split into variable costs and fixed costs. Variable costs are those related to fuel and to maintenance and repairs. Fuel consumption is assumed to be proportional to total kilometers driven, $m_t q_t$. Let $\omega_F > 0$ denote litters per kilometer (or gallons per mile in U.S. terminology), taken as constant for simplicity.[8] The lower ω_F, the larger fuel efficiency is. Fuel can be purchased at a certain price in an oil station, including a tax per litter of fuel, i.e.:

$$\left(p_t^F + \tau_t^F\right)\omega_F m_t q_t, \tag{6}$$

where p_t^F denotes the net price of fuel, and τ_t^F represents the fuel tax which it is considered as a lump-sum tax. F_t is the total expenditure of fuel. In a similar manner, vehicle maintenance and repairs are also assumed to be proportional to kilometers driven by a parameter $\omega_{MR} > 0$:

$$p_t^{MR}\omega_{MR} m_t q_t, \tag{7}$$

where p_t^{MR} denotes the price per unit of maintenance and repair services.

Denoting o_t the operating (variable) cost per kilometer, collected through expressions (6) and (7), we may write it in a compact notation:

$$o_t = \left(p_t^F + \tau_t^F\right)\omega_F + p_t^{MR}\omega_{MR}. \tag{8}$$

Operating costs are assumed to be proportional to driving distance. In practice, operating costs can also be affected by congestion when stop-and-go conditions occurs. Aggregating over the total variable utilization costs of vehicles gives

$$o_t m_t q_t. \tag{9}$$

Some other expenditures correspond to fixed costs. These expenditures, such as fees, tolls and

[8]Aghion et al. (2012) show evidence that automotive industry tend to innovate relatively more in fuel efficiency under tax-adjusted fuel prices increases. Wei (2013) uses a vintage capital within a DSGE model to incorporate innovations in fuel economy. We opt to take fuel efficiency ω_F as a parameter.

car insurances, do not depend on their use, but solely on their property:

$$p_t^{TI} q_t, \tag{10}$$

where p_t^{TI} denotes an aggregate price for these fixed costs.

Denoting x_t as the amount of new cars purchases, vehicles q_t are accumulated according to the following geometric form:

$$q_{t+1} = (1 - \delta_q)\, q_t + x_t, \tag{11}$$

where $0 < \delta_q < 1$ denotes cars' depreciation rate (including scrapped cars), and x_t represents units of brand new cars, which can be traded at a price $p_t^X \left(1 + \tau_t^X\right)$, with τ_t^X being an indirect tax on the acquisition price.

Savings from households are accumulated in a capital asset k_t, which is employed by firms to produce goods, rendering R_t units of income per unit of capital k_t. This asset is accumulated according to:

$$k_{t+1} = (1 - \delta_k)\, k_t + i_t, \tag{12}$$

where i_t accounts for household's gross investment and δ_k is the capital depreciation rate.

Finally, household's budget can be written as follows:

$$c_t + (1 + \tau_t^X) p_t^X x_t + i_t + \left(o_t m_t + p_t^{TI}\right) q_t = W_t \widetilde{h}_t + R_t k_t + \pi_t + TR_t, \tag{13}$$

where $(W_t,\, R_t)$ represent the hourly wage and the rental price of capital, respectively, π_t are firm profits, and TR_t are net lump-sum transfers.

2.2 Firms

We consider a representative profit maximizer firm that faces perfectly competitive markets of goods and factors. This firm produces a numeraire good Y_t, whose market price can be normalized to one. The firm hires capital and hours worked, (K_t, H_t), and exploits a positive externality from the public stock of capital, $K_{g,t}$. Resources are transformed into the final good using the following CES technology:

$$Y_t = A_t \left[\mu K_t^{\rho} + (1 - \mu) K_{g,t}^{\rho}\right]^{\alpha/\rho} \widetilde{H}_t^{1-\alpha}, \tag{14}$$

where A_t represents the total factor productivity[9], $0 < \alpha < 1$ is the capital income share of output, and $\mu \in (0,1)$ the weight on private capital relative to public road infrastructure. $\rho < 1$ determines the elasticity of substitution between private capital and road infrastructure. The degree of complementarity between K_t and $K_{g,t}$ increases as ρ tends to $-\infty$. The technology presents constant return to scale for $(K_t, H_t, K_{g,t})$. Note that profits are not zero in equilibrium, once we account by the non paid rents from using the public input $K_{g,t}$. These rents will be rebated to consumers via lump-sum transfers in budget (13).

2.3 The city hall

This city hall is headed by a mayor that must take decisions about taxes and the construction of road infrastructure. Tax revenues come from an ad valorem tax on fuel consumption τ_t^F, and a sale tax on new cars τ_t^X, levied on the price of new vehicles: $(1+\tau_t^x)\,p_t^x$. Fiscal revenues, FR_t, therefore amount to:

$$FR_t = \tau_t^F \omega_F m_t q_t + \tau_t^x p_t^x x_t. \tag{15}$$

The city hall uses these revenues to finance expenditure in highway investment $I_{g,t}$, and balances budget in every period:

$$FR_t = I_{g,t} + TR_t. \tag{16}$$

where $TR_t \gtrless 0$ are lump-sum transfers rebated to the households. We define an exogenous rule for public road investment:

$$I_{g,t} = \xi FR_t \tag{17}$$

where ξ, is the proportion of fiscal revenues devoted to road investment.

The stock of public input $K_{g,t}$ (roads) is accumulated according to

$$K_{g,t+1} = (1-\delta_g)\,K_{g,t} + I_{g,t}, \tag{18}$$

with δ_g being the road depreciation rate.

All in all, under the assumption that the set of prices $\left(p_t^X, p_t^F, p_t^{MR}, p_t^{TI}\right)$ are exogenously given

[9] It is assumed for A_t to follow a AR(1) stationary process such that:

$$\ln A_t = (1-\varsigma_A)\ln \bar{A} + \varsigma_A \ln A_{t-1} + \varepsilon_t^A,$$

where $\varepsilon_t^A \sim \mathcal{N}\left(0, \sigma_A^2\right)$ and $|\varsigma_A| < 1$.

in this economy, the particular feasibility constraint of our model can be expressed in the following terms:

$$C_t + I_t + I_{g,t} + p_t^X X_t + \left(p_t^F \omega_F + p_t^{MR} \omega_{MR}\right) M_t Q_t + p_t^{TI} Q_t = Y_t. \tag{19}$$

2.4 Competitive problem

The representative household maximizes the lifetime utility, subject to the budget constraint and the state equations for k_t and q_t :

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(c_t) + \psi_s \frac{s_t^{1-\gamma} - 1}{1-\gamma} - \psi_h \frac{(h_t + d_t)^{1+1/\upsilon}}{1+1/\upsilon} \right), \tag{20}$$

with respect to $(c_t, h_t, m_t, x_t, i_t, k_{t+1}, q_{t+1})$ and subject to

$$c_t + \left(1 + \tau_t^X\right) p_t^X x_t + i_t + \left(o_t m_t + p_t^{TI}\right) q_t = W_t \widetilde{h}_t + R_t k_t + \pi_t + TR_t, \tag{21}$$

$$k_{t+1} = (1 - \delta_k) k_t + i_t, \tag{22}$$

$$q_{t+1} = (1 - \delta_q) q_t + x_t. \tag{23}$$

where d_t is given by (4), and s_t is given by (2).

Summarizing, the first order conditions can be expressed as follows. First, a dynamic first order condition that determines the decision of investment in capital asset k_t:

$$\frac{1}{c_t} = \beta \mathbb{E}_t \left[\frac{1}{c_{t+1}} (1 - \delta_k + R_{t+1}) \right]. \tag{24}$$

Second, a static condition accounting for the leisure-consumption trade-off, given the real wage W_t. Note that this expression also incorporates the driving time d_t, as a by-product from the utilization of cars:

$$\varphi_h (h_t + d_t)^{1/\upsilon} = \theta \frac{s_t^{1-\theta}}{h_t^{1-\theta}} \frac{W_t}{c_t}. \tag{25}$$

Third, the decision of driving (kilometers traveled) is determined in a static condition, given the current state of vehicles q_t:

$$\varphi_s \phi m_t^{\phi(1-\gamma)-1} q_t^{-\gamma} = \frac{o_t - \phi(1-\theta) W_t h_t^{\theta} m_t^{\phi(1-\theta)-1} q_t^{-\theta}}{c_t}. \tag{26}$$

When the operating cost, o_t, or one of the components therein, increases, the household reduces the driving distance.[10]

Finally, the purchase of brand new vehicles (cars investment) is determined by the following dynamic equilibrium condition:

$$\begin{aligned} \frac{\left(1+\tau_t^x\right) p_t^x}{c_t} &= \beta \mathbb{E}_t \left[\left(1-\delta_q\right) \frac{\left(1+\tau_{t+1}^x\right) p_{t+1}^x}{c_{t+1}} + \right. \\ &\left. + \varphi_s m_{t+1}^{\phi(1-\gamma)} q_{t+1}^{-\gamma} + \left(\left(1-\theta\right) W_{t+1} \frac{h_{t+1}^{\theta}}{q_{t+1}^{\theta}} m_{t+1}^{\phi(1-\theta)} - o_{t+1} m_{t+1} - p_{t+1}^{TI} \right) \frac{1}{c_{t+1}} \right], \end{aligned} \tag{27}$$

The two last terms in this Euler equation can be viewed as the pricing kernel of automobiles, that is, the net gain from owning a car:

$$\Phi_t \equiv \underbrace{\varphi_s m_t^{\phi(1-\gamma)} q_t^{-\gamma} + \left(1-\theta\right) W_t \frac{h_t^{\theta} m_{t+1}^{\phi(1-\theta)}}{q_t^{\theta}} \frac{1}{c_t}}_{\text{Marginal gain}} - \underbrace{\left[o_t m_t + p_t^{TI}\right] \frac{1}{c_t}}_{\text{Marginal cost}}, \tag{28}$$

i.e. the marginal gain from one additional automobile minus the marginal opportunity cost. Iterating forward on this expression, we reach the following pricing expression:

$$\left(1+\tau_t^x\right) p_t^x \frac{1}{c_t} = \frac{1}{1-\delta_q} \sum_{j=1} \beta^j \left(1-\delta_q\right)^j \mathbb{E}_t \left[\Phi_{t+j}\right].$$

The problem of profit maximization takes the following structure:

$$\max_{\left(\widetilde{H}_t, K_t\right)} \left[\pi_t = Y_t - W_t \widetilde{H}_t - R_t K_t \right], \tag{29}$$

[10] When $\theta = 1$, kilometers traveled in expression (26) reduces to:

$$m_t = \left[\frac{\varphi_s \phi}{q_t^{\gamma}} \frac{c_t}{o_t} \right]^{\frac{1}{1-\phi(1-\gamma)}}.$$

Given the stock of vehicles q_t, the elasticity of kilometers driven with respect to the operating cost o_t is given by

$$\epsilon_{oc}^{km} = \frac{1}{1-\phi\left(1-\gamma\right)}.$$

This problem produces the following first order conditions:

$$W_t = (1-\alpha)\frac{Y_t}{\widetilde{H}_t}. \tag{30}$$

$$R_t = \alpha\frac{Y_t}{K_t}\frac{\mu K_t^{\rho}}{\mu K_t^{\rho} + (1-\mu) K_{g,t}^{\rho}}. \tag{31}$$

These two conditions state that firms will demand hours worked and capital so that the value of their marginal products will equate the hiring costs, W_t and R_t, respectively. Since $K_{g,t}$ is supplied by the government, it is considered to be exogenously given for firms.

Given these two expressions, unpaid rents from the public input K_g can be expressed as

$$\pi_t = \alpha Y_t \frac{(1-\mu) K_{g,t}^{\rho}}{\mu K_t^{\rho} + (1-\mu) K_{g,t}^{\rho}} > 0. \tag{32}$$

2.5 Equilibrium

Let $\zeta_t = (k_t, q_t)$ denote the vector of individual state variables. Given a government policy, $\{\tau_{t,}^F, \tau_t^X, I_{g,t}, TR_t\}$, a competitive equilibrium is a set of decision rules,

$$\{c(\zeta_t), x(\zeta_t), m(\zeta_t), h(\zeta_t), k(\zeta_{t+1})\},$$

aggregate choices,

$$\{C(\zeta_t), X(\zeta_t), M(\zeta_t), H(\zeta_t), K(\zeta_{t+1})\},$$

prices for fuel p_t^F, new vehicles p_t^X, maintenance and repairs p_t^{MR}, and tolls and cars insurances p_t^{TI}, and factor prices $W(\zeta_t)$ and $R(\zeta_t)$, such that

1. Given the government policy and factor prices, households' decisions are maximized (20), subject to the budget constraint (22), the state equations for capital (23), vehicle accumulation (11), and non-negativity constraints.

2. All factors (hours and capital) are hired at their marginal productivities: (30), (31).

3. The government satisfies its budget constraint (16) every period.

4. Markets clean: labor demand is equal to labor supply. Condition (12) holds for physical capital. The feasibility condition for the final good holds, (19).

5. The representative agent condition holds, i.e. aggregate choices coincide with individual ones when the latter is representative:

$$\begin{aligned}
K\left(\zeta_{t+1}\right) &= k\left(\zeta_{t+1}\right), \\
Q\left(\zeta_{t+1}\right) &= q\left(\zeta_{t+1}\right), \\
H\left(\zeta_{t}\right) &= h\left(\zeta_{t}\right), \\
C\left(\zeta_{t}\right) &= c\left(\zeta_{t}\right), \\
X\left(\zeta_{t}\right) &= x\left(\zeta_{t}\right).
\end{aligned}$$

3 Calibration

In order to examine the quantitative implications of our model, we need to assign a value to the different parameters. The set of (a total of 19) parameters of the model are the following:

$$\left(\beta, \delta_k, \varphi_h, \upsilon, \theta, \varphi_s, \phi, \gamma, \omega_F, \omega_{MR}, \delta_q, \alpha, \rho, \delta_g, \eta_0, \eta_1, A, \mu, \xi\right).$$

Additionally, the model includes two tax rates:

$$\left(\tau^F, \tau^x\right).$$

four exogenous prices:

$$\left(p^X, p^F, p^{MR}, p^{TI}\right).$$

and the parameters associated with the stochastic processes assumed for the shocks.

$$\varsigma_F, \sigma_F, \varsigma_A, \sigma_A$$

We may categorize them mainly as follows: parameters associated with preference specifications, parameters related to technology, parameters that relate to travel time and car use, and tax rates. Some of them can be set directly without solving the model, whereas others require the calculation of the steady-state to set its value as a function of a number of fundamental targets. Table 1 summarizes the set of targets for which the model is calibrated. Table 2 summarizes the calibrated values determined ex-ante (externally), whereas Table 3 summarizes calibrated values internally.

3.1 Model targets

Parameters of the model are calibrated to match a number of targets, which are summarize in table 1. Steady state output is normalized to one, $Y^* = 1$. Private gross investment over GDP in steady state (I^*/Y^*) is fixed to be 0.1766. The fraction of hours worked over total available discretionary time (which it is also normalized to 1) is 0.3333, using data from BEA and assuming a total available discretionary time of six days by week times 52 weeks by year times 16 hours by day (we assume that 8 hours are needed for sleeping). D^* is obtained as the fraction of hours driving. Duranton and Turner (2011) pointed out that in 2001, an average American household spent 161 person-minutes per day in a passenger vehicle (2.68 hours). Dividing by 24 hours a day, the resulting fraction of hours driving is:

$$D^* = \frac{2.68}{24} = 0.1117$$

Following Parry *et al*. (2014), delayed driving time is 107.3 hours per year, or:

$$Delay = \frac{107.3}{365 \times 24} = 0.0122$$

This implies that in steady state, a total of 1-0.3333-0.1117=0.5550 is leisure, defined as all other activities but working or driving and that approximately, about 11% of total driving time is due to congestion.

As of 1995, the National Household Travel Survey (NHTS) reports an estimate average of 1.33 workers per household and an average of 1.32 vehicles per worker. From these figures we obtain an absolute stock of vehicles per households of $Q^* = 1.33 \times 1.34 = 1.7822$. Parry *et al*. (2014) report an average of 18.5 thousand kilometers driven and hence we set $M^* = 18.5$. These figures implies a steady-state value for vehicle-kilometers traveled of $VKT = M^*Q^* = 1.7822 \times 18.5 = 32.9707$.

According to Eurostat database, which provides a detailed structure for transport expenditures per household; fuel expenditure accounts for 3.78% while maintenance and repairs expenditures account for 2.44% of consumption expenditure in 1999. The share of cars insurance expenditures, only available for 2005 and 2010, average value is of 1.5% of consumption expenditures. For the U.S., we use data from the BEA. Average fuel consumption over GDP is 0.0201, whereas maintenance and repairs represents a 1.62% of GDP. Finally, fixed costs including tolls and insurances over GDP is 0.007. Therefore, for the steady state conditions we choose the following targets:

$$\frac{F^*}{Y^*} = (p_F + \tau^F)\,\omega_F M^* Q^* = 0.0201, \tag{33}$$

$$\frac{Z^*}{Y^*} = p_{MR}\,\omega_{MR} M^* Q^* = 0.0162, \tag{34}$$

$$\frac{TI^*}{Y^*} = p_{TI}\,Q^* = 0.0070, \tag{35}$$

These figures implies that about 7.24 percent of GDP is expending in goods associated with cars use. Since p_{MR} is normalized to 1, we can state that ω_{MR} equals 0.0162 and that p_{TI} equals 0.007. Moreover, since p_F is set equal to 3.150 dollars per gallon, we obtain a value for ω_F equal to 0.0064. Also from the BEA we obtain that the fraction of new cars investment over GDP is 0.0291.

Finally, According to the International Transport Forum, investment in inland transport investment has remained constant, at about 0.8% of GDP (excluding Japan). Nevertheless, using data from BEA, average road government investment in the U.S. is 1.48% of GDP, that is $I_g^*/Y^* = 0.0148$.

Table 1: Model targets

Target	Notation	Value	Source
Output (GDP)	Y^*	1.0000	Normalization
Fraction of hours worked	H^*	0.3333	BEA
Fraction of hours driving	D^*	0.1117	Duranton and Turner (2011)
Private gross investment over GDP	I^*	0.1766	BEA
New cars investment over GDP	$p^x X^*$	0.0291	BEA
Fuel expenditures over GDP	F^*	0.0201	BEA
Maintenance and repairs over GDP	Z^*	0.0162	BEA
Fixed costs (tolls and insurances) over GDP	TI^*	0.0070	BEA
Gross stock of passenger vehicles	Q^*	1.7822	NHTS
Number of kilometers driven	M^*	18.5000	Parry et al. (2014)
Public gross ROAD investment over GDP	I_g^*	0.0148	BEA

Table 2: Ex-ante calibrated parameters

Category	Notation	Parameter definition	Value	Source
Preferences	υ	Frisch elasticity of labor supply	0.7200	Heathcote *et al.* (2010)
	θ	Fischer complementarity hours-cars	0.9280	Fischer (2007)
	β	Discount factor	0.9902	Real interest rate of 4%
Technology	α	Capital income share	0.3300	Gollin (2002)
	ρ	Complementarity capital-to-road	-0.5000	Ad hoc
	δ_k	Capital depreciation rate	0.0150	BEA
	δ_g	Road depreciation rate	0.0150	BEA
Travel and car use	η_1	Elasticity of driving time wrt congestion	2.5000	Parry *et al.* (2014)
	δ_x	Cars depreciation rate	0.0208	BEA
Exogenous prices	p^F	Price of fuel	3.1500	EIA
	p^{MR}	Price of maintenance and repairs	1.0000	BEA
Tax rates	τ^F	Fuel (ad valorem) tax	0.4950	EIA
	τ^x	VAT tax pm cars' purchase price	0.0670	BEA
	τ^c	Consumption tax	0.0600	BEA
Exogenous shocks	ς_A	Persistence of TFP shocks	0.9517	Estimation
	σ_A	Standard deviation of TFP shocks	0.0075	Estimation
	ς_F	Persistence of fuel price shocks	0.9590	Estimation
	σ_F	Standard deviation of TFP shocks	0.0676	Estimation

3.2 Parameters calibrated externally

Preferences $(\upsilon, \theta, \beta)$: First, the calibrated value for the Frisch elasticity, υ, is set equal to 0.72 according to Heathcote *et al.* (2010). Chetty *et al.* (2011) obtained a similar value using micro data and considering a married couple as the notion of household. This value is consistent with the micro evidence since we assume stable individuals, see Dyrda *et al.* (2012). The parameter representing the complementarity between hours and cars, θ is set equal to 0.928 based on the correlation between national income and new cars investment data from the Bureau of Economic Analysis, as shown by Fischer (2007). From the steady state conditions from the model we obtain that:

$$\beta = \frac{1}{1 - \delta_k + R^*}, \tag{36}$$

The discount factor, β takes a value of 0.9902, a value standard in the literature when considering quarterly data. This value is selected to match a value for the real interest rate net of depreciation of 4% annually.

Technology $(\alpha, \rho, \delta_k, \delta_g)$: Capital income's share of output, α, is set equal to 0.35. This parameter can be measured with data from the National Accounts. This share is consistent with those provided by Gollin (2002), who estimated that the capital income share should be within the [0.2, 0.35] interval in a wide set of countries under consideration. For the U.S. he reported a range from 0.257 to 0.396. Roads are used to commute and transport things. In the literature, it is standard to assume a Cobb-Douglas production function with both public and private capital. Examples are Finn (1993), Guo and Lansing (1997), and Cassou and Lansing (1998), as they consider both types of capital assets as homogenous, and hence, the elasticity of substitution between public and private capital is unitary. Nevertheless, here we relax that assumption, as we consider a complementarity relationship between private capital and the stock of roads. Thus, ρ is set equal to -0.5 as a benchmark to reflect some complementarity between capital and roads. Roads depreciates due to weather conditions but mainly by heavy truck traffic. Capital stock depreciation rate is assumed to be 0.02. Depreciation rate for road infrastructure, δ_g, is calculated according to OECD methodology in OECD (2001) with a value set equal to 0.015 (this implies a 4,43% depreciation rate annually). Boskin et al. (1989) assume that roads depreciate geometrically at a rate of 1.98 percent per year.

Travel and car use (η_1, δ_q): Following Parry *et al.* (2014), we set a value for η_1 equal to 4, which according to Small and Verhoef (2007) is a value consistent with the Bureau of Public Roads formula, used traditionally to predict vehicle speed. Empirical studies suggest that this value is in the range 2.5 to 5, depending on the size of the urban centre considered. The larger the urban centre, the higher the value.[11] Depreciation rate for cars is calculated by assuming an average lifespan of 12 years, which result in a quarterly depreciation rate, δ_q equal to 0.0208 (0.053 in annual terms 8.33 percent yearly).

$$\delta_q = \frac{1}{4 \times 12} = 0.0208. \tag{37}$$

Wei (2013) set the maximum life span of vehicles to be equal to 15 years and then the depreciation rate is set at an annual rate of 0.1.

Exogenous prices (p^F, p^{MR}): We consider prices related to fuel consumption and maintenance

[11] See Vickrey (1963), Small and Verhoef (2007) and Small (1992).

and repairs of cars as exogenous given. The other two prices are calibrated endogenously. We normalize the price of maintenance and repairs, p^{MR} to 1. The price of fuel, p^F, is fixed to 3.15, following data from the Energy Information Agency (EIA).

Tax rates (τ^F, τ^x, τ^c): The fuel tax rate, τ^F, is set equal to 0.495, following data from the Energy Information Agency (EIA). The tax on cars' purchase price is fixed to be 20%.

Exogenous shocks $(\varsigma_A, \sigma_A, \varsigma_F, \sigma_F)$: Parameters driving the AR(1) processes defined for both TFP and oil price shocks are estimated by OLS. For the oil shock, we use data from table 2.3.4. from BEA. We take line 11: Gasoline and other energy goods, as the reference series to estimate oil price shocks. Data has a quarterly frequency and the sample period is 1970:Q1 to 2014:Q4. Estimated values are 0.986 for the persistence parameters of the AR(1) process, ς_F, and 0.071 for the standard deviation, σ_F.

3.3 Parameters calibrated internally

Next, we calibrate the parameters that require solving the model. We summarize them in Table 3. The preference parameter representing the willingness to work, φ_h, is chosen to match average household hours worked in the market $H^* = \frac{8}{24} = \frac{1}{3}$, given the steady state value of consumption, $C^*/Y^* = 0.7362$, and driving time.

Parry *et al.* (2014) estimate a surface road capacity of 17.3 km. per vehicle. Given that the value for Q^* is 1.7822, this implies that the stock of roads is 30.8321 kilometers per household. Measuring the stock of roads in term of output, given the value for road investment and road depreciation rate, in steady state the stock of road is calculated to be:

$$K_g^* = \frac{I_g^*}{\delta_g} = \frac{0.0148}{0.015} = 0.9867. \tag{38}$$

Similarly, the steady state stock of capital is given by:

$$K^* = \frac{I^*}{\delta_k} = \frac{0.1766}{0.015} = 11.7730. \tag{39}$$

Following Bento *et al.* (2009), for all households and cars, the average elasticity, ϵ_{oc}^{km}, is set equal to 0.74. It is lower for new cars than for older vehicles. Therefore, the parameters measuring the decreasing marginal returns from vehicle services, ϕ, and the parameter measuring the constant relative risk aversion for car services, γ, are related such as:

$$\epsilon_{oc}^{km} = \frac{1}{1-\phi\,(1-\gamma)} = 0.74 \tag{40}$$

Estimates of relative risk aversion are commonly set between one and two. From the steady state equilibrium conditions for driving decision and car investment decision, derived from equations (26) and (27) we obtain that the following expression for the price of cars:

$$(1+\tau^{x})\,p^{x} = \frac{\beta}{\phi}\frac{(1-\phi)\,oM - \phi p^{TI}}{1-\beta\,(1-\delta_q)} \tag{41}$$

Solving for ϕ in the above expression, we obtain a value of 0.4077. Therefore, γ is set equal to 1.573 and φ_s is set equal to 0.123 which also matches the fraction of new cars investment over GNP, equal to 0.04 according to data from the Bureau of Economic Analysis:

$$\frac{p^{x}X^{*}}{Y^{*}} = 0.0291$$

Given the calibrated value for cars depreciation rate ($\delta_q = 0.0208$), from steady state equation we obtain that:

$$X^{*} = Q^{*}\delta_q = 0.0208, \tag{42}$$

and given Q^{*}, this results in:

$$p^{x} = \frac{0.0291}{0.0208} = 1.3999 \tag{43}$$

The remaining price, the price of tolls and insurance, is also calibrated internally using the following expression:

$$p^{TI} = \frac{TI^{*}}{Q^{*}} = 0.0066$$

According to these assumptions, the steady state condition derived from equation (25) gives:

$$\varphi_h = (1-\alpha)\,\theta\frac{Y^{*}}{C^{*}}\frac{1}{H^{*}\,(H^{*}+D^{*})^{1/\upsilon}} = 6.4221. \tag{44}$$

From the steady state condition for VKT of the model we also obtain that:

$$\varphi_s = \frac{o_t - \phi(1-\theta)W_t H^{\theta} M^{\phi(1-\theta)-1} Q^{-\theta}}{\phi M^{\phi(1-\gamma)-1} Q^{-\gamma} C} = 0.1209. \tag{45}$$

Given our definition of congestion, driving time is defined using the standard power function:

$$D^* = \eta_0 \left(\frac{M^* Q^*}{K_g^*} \right)^{\eta_1} = 0.1677 \tag{46}$$

From that expression, we compute the value for the scale parameter η_0, given that $M^*Q^* = 1$, $K_g^* = 0.5333$, and $\eta_1 = 2.5$, we obtain a value of 0.1619. Note that our measure of congestion in steady state is just $1/0.9867$ =1.013.

Consumption of gasoline liters per kilometer is calculated as:

$$\omega_F = \frac{F^*}{((p^F + \tau^F)M^*Q^*)} = 0.0055.$$

Similarly, the proportionality parameters of vehicle maintenance and repairs to kilometers driven is calculated as:

$$\omega_{MR} = \frac{Z^*}{(p^{MR}(1+\tau^c)M^*Q^*)} = 0.0153.$$

Given these parameters, the operating cost per kilometer driven is calculated to be o =0.0363.

Capital income over total income in steady state is given by:

$$\frac{R^*K^*}{Y^*} = \alpha \frac{\mu (K^*)^{\rho}}{\mu (K^*)^{\rho} + (1-\mu)\left(K_g^*\right)^{\rho}}, \tag{47}$$

From that expression we calculate the value for $\mu = 0.9463$.

Finally, Total Factor Productivity is obtained as a residual from the aggregate production function as:

$$A^* = \frac{Y^*}{\left[\mu (K^*)^{\rho} + (1-\mu)\left(K_g^*\right)^{\rho}\right]^{\alpha/\rho} \left(\widetilde{H}_t^*\right)^{1-\alpha}} = 1.0334.$$

Table 3: **Internally calibrated parameters**

Category	Notation	Parameter definition	Value
Preferences	ψ_h	Willingness to work	6.4221
	ψ_s	Willingness to drive	0.1209
Technology	μ	CES weight for private capital	0.9463
	A	Total Factor Productivity	0.8925
Travel and car use	ω^F	Gasoline liters per kilometer	0.0055
	ω^{MR}	Cars maintenance and repairs	0.0153
	η_0	Congestion scaling parameter	0.1619
	ϕ	Diminishing returns of car use	0.4676
	γ	CRRA for car use	1.7513
	o	Operating cost per km. driven	0.0363
Prices	p^x	Price of cars	1.3999
	p^{TI}	Price of tolls and insurance	0.0066
Road investment	ξ	Fraction of fiscal income to road	1.0000

4 Model Economy Dynamics

This section shows the dynamics of the model economy via impulse-response functions to TFP shocks.

Our first exercise considers the case of an exogenous positive neutral shock to the economy, that is, an increase in Total Factor Productivity, A_t. This is a standard shock studied in most real business cycle models and so it is used as a benchmark. The idea is to study how the economy reacts to a productivity shock when traffic, cars investment, driving time, and public investment in road are added to the standard model. First, we conduct the analysis conditioned to the fact that road public investment is a fixed proportion of fiscal revenues. This implies that a positive productivity shock increases fiscal revenues (this is always true, as we only consider value added taxes or lump-sum taxes). As fiscal revenues increases, road investment also increases. Next, we isolate the effects of the productivity shock to the case in which the road stock is fixed. Dynamic responses of the variables are similar in both contexts.

We assume that TFP increases by one standard deviation on impact. As expected, this shock raises output on impact, as more output is produced for given factor inputs. Private investment also increases in the period when the shock occurs given that the shocks reduce the marginal cost of capital accumulation. As a consequence, the private capital stock also increases given the rise in its productivity, increasing the persistence of output to the shock. These changes in output and physical capital lead to a gradual increase in consumption above its steady state. Public investment in road is assumed to increases as a fixed proportion of fiscal revenues is assumed to be expended on road investment. Thus, the overall effects of this shock in our theoretical framework are the same (from a qualitative point of view) than in the standard real business cycle model without traffic and roads: a rise on output, investment, consumption and capital stock in response to the shock.

Nevertheless, the inclusion of roads and traffic in the households utility function and in the aggregate production function allows us to move beyond the standard results. The productivity shock also impacts positively on VKT, and initially the level of congestion increases, rising the travel time. Nevertheless, congestion is reduced below the steady state after some periods, as a direct consequence of the larger stock of roads. The rise of the VKT is observed in spite that the flow kilometers driving, M, decreases, given that the current stock of cars, Q, increases more than proportionally.

The impact of an increase in TFP on the macroeconomic variables is summarized in Figure 1. We plot deviations in percentage points from the steady state values. The shock has a positive effect on output, as expected, and described above. Both consumption and private investment increase on impact but through different channels. On the other hand, the shock increases the rental rate of capital, thus increasing private investment. Labor services increase in impact, but after some period the response is negative. This is provoked by a negative response of working hours.

Figure 2 summarized the impulse-response functions for the variables related to cars use. The number of kilometers traveled increases in impact over its steady state value although after some periods the behavior turn out to be negative. The rise in cars investment transforms in a rise in the stock of cars. Given the response of these two variables, the vehicle-kilometers traveled also increases provoking a rise in congestion. Importantly, it is this rise in road congestion the factor behind the reduction in both working hours and leisure.

As a conclusion, the model predicts a rise in road congestion as a consequence of a positive aggregate productivity shock. The higher level of traffic is produced initially by a larger value of kilometers driven but afterward by the rise in the stock of cars. Nevertheless, the higher level of road congestion has a negative impact on the economy, mitigating the positive effects of the productivity shock. In other words, more time is spent in driving, reducing the available time for both working activities and leisure.

5 Road investment and generated traffic: The fundamental law of road congestion

In principle, one might think that congestion can be solved by investing in new roads as capacity is used. Nevertheless, as have been pointed out by Downs (1962) as road capacity increases, traffic also increases in the same proportion as to congestion remains constant. This is the so-called "Fundamental Law of Road Congestion". This law was initially suggested by Downs (1962, 1992) and basically says that vehicle kilometers traveled increases proportionately to roadway lane kilometers for highways. This implies that it is not possible to reduce congestion by building more roads as more traffic is generated in response to the new road capacity. There is a vast empirical literature studying the relationship between road capacity expansion and traffic. One important result in this literature is that road capacity expansions cannot reduce congestion. New roads generate additional traffic given the existence of a latent demand. This generated traffic reduces or even cancel out the effects of construction new road capacity as an instrument to reduce congestion. Generated traffic in response to road capacity expansion comes from two sources (Litman, 2016): Diverted traffic and induced traffic. Diverted traffic is due to trips shifted in time, route and destination, what Downs, 1992, call the *Triple Convergence.* Induced traffic is related to shifts from other modes, longer trips and new trips. Jorgensen (1947) was the first to estimate empirically the response of traffic to road capacity expansions for the case of the construction of the Merritt and Wilbut Cross parkways in the corridor between New York city and New Haven, Connecticut. The Fundamental Law of Traffic Congestion has an important economic policy recommendation: It is not possible to relieve road congestion by building more roads. In fact, highway construction is expensive and the value of the elasticity of traffic with respect to road improvements is a central question in the debate about new road construction. Based on that Fundamental Law it results clear that generated traffic reduces benefits of road capacity expansion over congestion.

Nevertheless, estimated generated traffic associated to road improvements are very imprecise and estimated elasticity goes from 0 to 100 percent. In general, the majority of empirical estimations only support partially the "fundamental law of highways congestion". For instance, Hansen (1995) obtains values in the rage from 0 to 0.3 one year after improvement and from 0.2 to 0.8 four years later. Using data from the California Metropolitan Areas, he obtain that a the county level a 1 percent increase in lane-miles induces an immediate 0.2 percent increase in traffic, building to a 0.6 percent increases two years after the improvement. Similar values are obtained at the metropolitan level. However, he estimated that only a small fraction of the rise in VKT is due to the increase in lane-miles. Cervero (2003), also using data from the California freeway estimates a long-term elasticity of 0.64. Although estimated elasticity is always positive, they are usually below one.[12] Nevertheless, a recent article by Duranton and Turner (2011) estimated the elasticity of vehicle kilometers traveled with respect to lane kilometers, obtaining a value close to one for a number of alternative specifications. This is an empirical evidence in favor of the "fundamental law of highways congestion" as proposed by Downs (1962). Moreover, they suggest that the fundamental law can be extended to a broad class of major urban roads, resulting in a general "fundamental law of road congestion". Based on these results, Duranton and Turner (2011) conclude that road capacity expansion is not an appropriate policy to reduce traffic congestion and that the correct instrument should be congestion pricing. Using a similar approach, Hsu and Zhang (2014) obtain an estimated elasticity between 1.24 and 1.34 for Japan.

Interestingly, Duranton and Turner (2011) do an accounting exercise in order to quantify the relative contribution of each possible sources for the increase in traffic following a road capacity expansion. They consider four sources: Changes in trucking and commercial driving; changes in household driving decision; changes in population; and diversion of traffic. The obtain that between 19 and 29% of total traffic increase is due to trucks and that migration accounts for between 5% and 21%. For the relative importance of changes in household driving decision, they obtain a contribution between 9 and 39% of total traffic increase. Finally, diversion of traffic accounts for between 0 and 10% of total traffic increase.

In this Section we evaluate what the model says about the "fundamental law of road congestion". For that, we introduce a shock in the road investment decisions from the government. In the model, road investment is an exogenous decision taken by the government. In the central planner problem (what we call the mayor problem), the stock of road is chosen just to maximize social welfare. The exercise will consist in calculating the response of traffic to the rise in the stock of roads. Our model only include two out of the four sources of traffic increase considered by Duranton and Turner (2011), that is, commercial driving and household driving decisions and therefore, we would expect an increase in traffic lower than the increase in road capacity.

Figure 3 plots the dynamics of the variables following a rise in the proportion of GDP devoted

[12] Cervero and Hansen (2002). Cervero (2002).

to road construction. We assume that road investment increases by 10%. This rise in road investment implies that in the long-run, the stock of roads (our variable for road capacity) will rise also a 10%. We find that the increases in VMT is around 1/3 of the rise in the stock of road. Therefore, the model is able to generate a rise in vehicle-kilometers traveled as a consequence of the increases in lane kilometers of road. The response that we find is very robust to the parameters values.[13] Following Duranton and Turner (2011) results, the sum of commercial plus household driving decision account between 28 and 68%. Our estimated response is close to the lower value of that range.

The rise in VMT in the long-run is based on the rise in the number of cars but not in the number of kilometers driven. In impact, the kilometers driven rises, but as the stock of cars increases, kilometers driven reduces to the previous steady state level. That is, kilometers driven is not a function of the stock of roads, but road capacity determines the stock of cars in the long-run. Given that the increases of VMT generates by the model is less proportional with respect to the stock of roads, the level of congestion also reduces, as the model does not considered additional factors affecting induced traffic such as changes in population or diverted traffic.

Two important results derive from the analysis. First, the rise in VMT computed is related to the level of economic activity. In the model, the stock of road is an additional input in the production function. The rise in road investment decision by the government increases output in the long-run in about 4.5%, and hence, the implicit multiplier of public spending in highways is around 0.5. This result is consistent with the findings by Fernald (1999). Second, isolating

6 Optimal traffic density and optimal congestion: Mayor's problem

The welfare effects of traffic congestion has been thoroughly analyzed in the literature. Drivers do not take into their own effect on congestion when taking driving decisions. As they only cares about their own costs, efficiency requires a Pigouvian tax equal to the gap between the marginal cost to all drivers and the cost for the individual. Seminal paper is those of Vickrey (1963, 1969). This literature has focused on the used of optimal tolls as an optimal instrument to solve congestion problem. Examples are Arnott et al. (1990, 1993, 1994).

Walters (1961)

[13] To chek the robustness of the model we calculate the VMT responses to road change as a function of the main parameter of the model. We change the values of two key parameters, the elasticity of VMT with respect to operating costs and the elasticity of driving with respect to road traffic congestion. We vary each parameters one at a time. VMT responses to road capacity expansion remains constant.

In this section we study the level of pigouvian taxes.

Given the response of traffic to road capacity expansion, the effect of road investment on congestion is very limited or, as pointed out by Duranton and Turner (2011) even null, and the only space to alleviate congestion problem is via congestion pricing policies.

Parry and Small (2005), taking into account all external costs of traffic (congestions, accidents, and air pollution), obtain that the optimal gasoline tax in the U.S. is 1.01 dollars per gallon, and 1.34 dollars per gallon for the U.K.

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However, as Parry, Walls and Harrington (2007) pointed out, a fuel tax rise driving costs for all regions at all times being a very blunt instrument for alleviating traffic congestion which is very specific to rush hour periods in urban ares. Instead, the ideal instrument is a road-specific congestion toll that varies with time a day and advances in electronic metering technology make it feasible.

Congestion is considered as a negative externality. In our model, we have two taxes: vehicles sales tax and fuel tax. We study the optimal tax for each case.

Parry et al. (2014) calculate travel delays costs by estimating the relationship between travel delays and different transportation indicators and using evidence of the relationship between wages and how people value travel time.

To evaluate the cost of congestion, we first solve the social planner problem in our economy with cars, roads and traffic. The Lagrangian associated to the household's problem is given by

$$
\begin{aligned}
\mathcal{L} \;=\; & \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln\left(C_t\right) + \psi_s \frac{M_t^{\phi(1-\gamma)} Q_t^{1-\gamma} - 1}{1-\gamma} - \psi_h \frac{\left(H_t + D_t\right)^{1+1/\upsilon}}{1+1/\upsilon} \right), \\
& -\vartheta_{1,t} \left[C_t + p_t^X X_t + I_t + I_{gt} + \left(\left(p_t^F \omega_F + p_t^{MR} \omega_{MR} \right) M_t + p_t^{TI} \right) Q_t - Y_t \right] \\
& -\vartheta_{2,t} \left[K_{t+1} - \left(1-\delta_k\right) K_t - I_t \right] \\
& -\vartheta_{3,t} \left[Q_{t+1} - \left(1-\delta_q\right) Q_t - X_t \right] \\
& -\vartheta_{4,t} \left[K_{g,t+1} - \left(1-\delta_g\right) K_{g,t} - I_{g,t} \right]
\end{aligned}
$$

with respect to $(C_t, H_t, M_t, X_t, I_t, I_{gt}, K_{t+1}, Q_{t+1}, K_{g,t+1})$, where

$$
\begin{aligned}
Y_t &= A_t\left[\mu K_t^{\rho} + (1-\mu)K_{g,t}^{\rho}\right]^{\alpha/\rho}\widetilde{H}_t^{1-\alpha}, \\
\widetilde{H}_t &\equiv H_t^{\theta}Q_t^{1-\theta}, \\
D_t &\equiv \eta_0\left(\frac{M_tQ_t}{K_{g,t}}\right)^{\eta_1}.
\end{aligned}
$$

The first order condition for hours worked and investment are given by:

$$
\begin{aligned}
\varphi_h\left(H_t+D_t\right)^{1/\upsilon} &= \theta\left(1-\alpha\right)\frac{Y_t}{C_tH_t}, \\
\frac{1}{C_t} &= \beta\mathbb{E}_t\left[\frac{1}{C_{t+1}}\left[1-\delta_k+\alpha\frac{Y_{t+1}}{K_{t+1}}\frac{\mu K_{t+1}^{\rho}}{\mu K_{t+1}^{\rho}+(1-\mu)K_{g,t+1}^{\rho}}\right]\right].
\end{aligned} \tag{48}
$$

These two expressions do not differ from their competitive counterparts.

A key distinction in the social planner assignment arise in the first order conditions for kilometers driven and vehicles stock, which now internalize the cost driving time due to congestion. From expression, kilometers travelled is given by:

$$
\begin{aligned}
&\varphi_s\phi M_t^{\phi(1-\gamma)-1}Q_t^{1-\gamma}-\varphi_h\left(H_t+D_t\right)^{1/\upsilon}\eta_0\eta_1\frac{Q_t}{K_{g,t}}\left(\frac{M_tQ_t}{K_{g,t}}\right)^{\eta_1-1} \\
=\ &\left(p_t^F\omega_F+p_t^{MR}\omega_{MR}\right)\frac{Q_t}{C_t}-\phi(1-\alpha)(1-\theta)\frac{Y_t}{C_tM_t}.
\end{aligned} \tag{49}
$$

The equilibrium condition corresponding to vehicle investment is given by

$$
\begin{aligned}
\frac{p_t^X}{C_t} = \beta\mathbb{E}_t&\left[\frac{p_{t+1}^X}{C_{t+1}}\left(1-\delta_q\right)+\varphi_sM_{t+1}^{\phi(1-\gamma)}Q_{t+1}^{-\gamma}\right. \\
&-\varphi_h\left(H_{t+1}+D_{t+1}\right)^{1/\upsilon}\eta_0\eta_1\frac{M_{t+1}}{K_{g,t+1}}\left(\frac{M_{t+1}Q_{t+1}}{K_{g,t+1}}\right)^{\eta_1-1} \\
&\left.-\frac{1}{C_{t+1}}\left[\left(p_{t+1}^F\omega_F+p_{t+1}^{MR}\omega_{MR}\right)M_{t+1}+p_{t+1}^{TI}-\left(1-\theta\right)\left(1-\alpha\right)\frac{Y_{t+1}}{Q_{t+1}}\right]\right].
\end{aligned} \tag{50}
$$

Finally, the first order condition which determines the optimal amount of road stock is given by the following dynamic Euler expression:

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left\{ \varphi_h \left(H_{t+1} + D_{t+1}\right)^{1/v} \eta_1 \frac{D_{t+1}}{K_{g,t+1}} + \frac{1}{C_{t+1}} \left[1 - \delta_g + \alpha \frac{Y_{t+1}}{K_{g,t+1}} \frac{\left(1-\mu\right) K_{g,t+1}^{\rho}}{\mu K_{t+1}^{\rho} + \left(1-\mu\right) K_{g,t+1}^{\rho}} \right] \right\}. \tag{51}$$

6.1 Pigouvian taxation

Consider the conditions that determine the driving decisions under a competitive environment and under a centralized framework, expressions (26) and (49), respectively. Subtracting both conditions, we reach the following expression for the Pigouvian fuel taxation:

$$\tau_t^F = \frac{\varphi_h \left(H_t + D_t\right)^{1/v}}{1/C_t} \eta_1 \frac{D_t}{\omega_F Q_t M_t}. \tag{52}$$

Hence, the Pigouvian tax internalizes the external cost of road congestion, given in the right hand side of (52). The higher the willingness to work (φ_h) and the fuel efficiency (i.e. the lower ω_F), the higher the taxation required on fuel. Interestingly, the optimal tax depends on the parameter η_1.

Assume next that the fuel tax is fixed according to (52). Then, *under this assumption*, comparing the expressions that determine the new cars purchases under a competitive environment (27), and under a centralized framework (51), the Pigouvian sale tax on new cars purchases should be set according to the following rule:

$$\tau_t^X \frac{p_t^X}{C_t} = \beta \left(1 - \delta_q\right) \mathbb{E}_t \left[\tau_{t+1}^X \frac{p_{t+1}^X}{C_{t+1}} \right]. \tag{53}$$

This implies a Pigouvian sale tax associated to congestion equals to zero at any time t:

$$\tau_t^X = 0. \tag{54}$$

Note that this implies that the sale tax τ_t^X does not help internalize the external cost of congestion. The economic intuition behind this result is simple: congestion is related to the number of drivers using a road at a given moment of time. Therefore, it is related to the use of cars and not to the number of brand new cars and then, a sale tax is not an adequate tool for internalizing congestion costs.

Walters (1961) estimated an optimal fuel tax of a minimum of 33 cents per gallon. This value is large compared with the state and federal taxes of about nine or ten cents per gallon. Our calibrate fuel ad valorem tax is 0.495, that is, about five times the fuel tax in the Walters (1961) study. Parry and Small (2005) argue that congestion costs range between 1.5 cents and 9.0 cents per mile, with 3.5 cents being the assumed central marginal cost of congestion. Harrington (2006) propose a value of 6.5 cents per mile. On the other hand, *Monthly Energy Review* (November 2016, Table 1.8 Motor Vehicle Mileage, Fuel Consumption, and Fuel Economy), compiles an average miles per gallon of 17.40. This referenced figure includes all type of vehicles (passengers vehicles, vans, pickup trucks, sport utility vehicles and heavy-duty trucks). Combining these estimations, it results that the interval for the fuel tax that internalizes the external cost of congestion is in the range of 0.261 and 1.566 dollars per gallon, with a central value of 60.9 cents per gallon.

7 Conclusions

This paper develops a Dynamic Stochastic General Equilibrium model in which personal transport activities are included in the households utility function, road infrastructure in the production function, and road investment in the government decision problem. Households driving decision enters in the households utility function in two ways. First, households receives services from driving cars. Second, driving time is related to working time and labor is measured in efficient units where cars are considered as a working tools which complement labor hours. Moreover, public investment decisions are transformed in public capital (roads) stock. Finally, road infrastructure is considered as an additional input in the aggregate production function.

First, we examine the business cycle properties of the model and the relationship between traffic and economic variables. We find that traffic and hence congestion are pro-cyclical variables affecting the effects of a productivity shock on the economy. As driving time is considered as waste time (it is a disutility), congestion reduces the positive effect of a positive productivity shock (and the opposite).

Second, the model can be used to study the validity of the so-called "Fundamental Law of Road Congestion".

Finally, we use the model to obtain an expression for the optimal fuel tax.

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Appendices

A First order conditions that characterize descentralized equilibrium

In a descentralized economy the Lagrangian associated to the household's problem is given by

$$\begin{aligned} \mathcal{L} &= \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(c_t) + \varphi_s \frac{s_t^{1-\gamma}-1}{1-\gamma} - \varphi_h \frac{(h_t+d_t)^{1+1/\upsilon}}{1+1/\upsilon} \right), \qquad \text{(A.1)} \\ & -\lambda_{1,t}\left[c_t + \left(1+\tau_t^X\right)p_t^X x_t + i_t + \left(o_t m_t + p_t^{TI}\right)q_t - W_t\widetilde{h}_t - R_t k_t - \pi_t - TR_t\right] \\ & -\lambda_{2,t}\left[k_{t+1} - (1-\delta_k)k_t - i_t\right] \\ & -\lambda_{3,t}\left[q_{t+1} - (1-\delta_q)q_t - x_t\right], \end{aligned}$$

with respect to $(c_t, h_t, m_t, x_t, i_t, k_{t+1}, q_{t+1})$, where

$$s_t = m_t^{\phi} q_t, \qquad \text{(A.2)}$$

$$d_t = \eta_0 \left(\frac{M_t Q_t}{K_{g,t}}\right)^{\eta_1}, \qquad \text{(A.3)}$$

$$\widetilde{h}_t = h_t^{\theta} s_t^{1-\theta} = h_t^{\theta} m_t^{\phi(1-\theta)} q_t^{1-\theta}, \qquad \text{(A.4)}$$

$$o_t = \left(p_t^F + \tau_t^F\right)\omega_F + p_t^{MR}\omega_{MR}. \qquad \text{(A.5)}$$

The first order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} : \frac{1}{c_t} - \lambda_{1,t} = 0, \qquad \text{(A.6)}$$

$$\frac{\partial \mathcal{L}}{\partial h_t} : -\psi_h (h_t+d_t)^{1/\upsilon} + \lambda_{1,t}\theta \frac{s_t^{1-\theta}}{h_t^{1-\theta}} W_t = 0, \qquad \text{(A.7)}$$

$$\frac{\partial \mathcal{L}}{\partial x_t} : -\lambda_{1,t}(1+\tau_t^x)p_t^x + \lambda_{3,t} = 0, \qquad \text{(A.8)}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} : \varphi_s \phi m_t^{\phi(1-\gamma)-1} q_t^{1-\gamma} - \lambda_{1,t}\left[o_t q_t - \phi(1-\theta) W_t h_t^{\theta} m_t^{\phi(1-\theta)-1} q_t^{1-\theta}\right] = 0, \qquad \text{(A.9)}$$

$$\frac{\partial \mathcal{L}}{\partial i_t} : -\lambda_{1,t} + \lambda_{2,t} = 0, \qquad \text{(A.10)}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : -\lambda_{2,t} + \beta \mathbb{E}_t\left[\lambda_{1,t+1}R_{t+1} + \lambda_{2,t+1}(1-\delta_k)\right] = 0, \qquad \text{(A.11)}$$

and

$$\frac{\partial \mathcal{L}}{\partial q_{t+1}} : -\lambda_{3,t} + \beta \mathbb{E}_t \Big[\lambda_{3,t+1} (1-\delta_q) + \varphi_s m_{t+1}^{\phi(1-\gamma)} q_{t+1}^{-\gamma} \tag{A.12}$$

$$-\lambda_{1,t+1} \left[o_{t+1} m_{t+1} + p_{t+1}^{TI} - (1-\theta) W_{t+1} \frac{h_{t+1}^{\theta}}{q_{t+1}^{\theta}} m_{t+1}^{\phi(1-\theta)} \right] \Big] = 0.$$

The maximization problem associated with the firm is given by:

$$\max \left\{ A_t \left[\mu K_t^{\rho} + (1-\mu) K_{g,t}^{\rho} \right]^{\alpha/\rho} \ \widetilde{H}_t^{1-\alpha} - W_t \widetilde{H}_t - R_t K_t \right\}, \tag{A.13}$$

with respect to $\left(\widetilde{H}, K\right)$. The first order conditions are given by expressions:

$$\frac{\partial \Pi}{\partial \widetilde{H}_t} \ : \ W_t = (1-\alpha) \frac{Y_t}{\widetilde{H}_t}. \tag{A.14}$$

$$\frac{\partial \Pi}{\partial K_t} \ : \ R_t = \alpha \frac{Y_t}{K_t} \frac{\mu K_t^{\rho}}{\mu K_t^{\rho} + (1-\mu) K_{g,t}^{\rho}}. \tag{A.15}$$

where the demand for inputs takes the usual conditions in a competitive economy: the firm will hire labor and capital such that their marginal productivity equates their rental prices, W_t and R_t. Profits (unpaid rents to public input) is given by:

$$\pi_t = \alpha Y_t \frac{(1-\mu) K_{g,t}^{\rho}}{\mu K_t^{\rho} + (1-\mu) K_{g,t}^{\rho}}. \tag{A.16}$$

B FOC that characterize mayor's problem

The Lagrangian associated to the household's problem is given by

$$\begin{aligned} \mathcal{L} &= \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) + \varphi_s \frac{S_t^{1-\gamma} - 1}{1-\gamma} - \varphi_h \frac{(H_t + D_t)^{1+1/\upsilon}}{1+1/\upsilon} \right), \qquad \text{(B.1)} \\ & -\vartheta_{1,t} \left[C_t + p_t^X X_t + I_t + I_{gt} + \left(\left(p_t^F \omega_F + p_t^{MR} \omega_{MR} \right) M_t + p_t^{TI} \right) Q_t - Y_t \right] \\ & -\vartheta_{2,t} \left[K_{t+1} - (1-\delta_k) K_t - I_t \right] \\ & -\vartheta_{3,t} \left[Q_{t+1} - (1-\delta_q) Q_t - X_t \right] \\ & -\vartheta_{4,t} \left[K_{g,t+1} - (1-\delta_g) K_{g,t} - I_{g,t} \right] \end{aligned}$$

with respect to $(C_t, H_t, M_t, X_t, I_t, I_{gt}, K_{t+1}, Q_{t+1}, K_{g,t+1})$, where

$$\begin{aligned} Y_t &= A_t \left[\mu K_t^\rho + (1-\mu) K_{g,t}^\rho\right]^{\alpha/\rho} \widetilde{H}_t^{1-\alpha}, && \text{(B.2)} \\ \widetilde{H}_t &\equiv H_t^\theta S_t^{1-\theta}, && \text{(B.3)} \\ S_t &= M_t^\phi Q_t, && \text{(B.4)} \\ D_t &\equiv \eta_0 \left(\frac{M_t Q_t}{K_{g,t}}\right)^{\eta_1}. && \text{(B.5)} \end{aligned}$$

The first order conditions for consumption and hours worked are given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \frac{1}{C_t} - \vartheta_{1,t} = 0, \quad \text{(B.6)}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} : -\varphi_h \left(H_t + D_t\right)^{1/\upsilon} + \vartheta_{1,t}(1-\alpha)\theta H_t^{\theta(1-\alpha)-1} S_t^{(1-\theta)(1-\alpha)} \quad \text{(B.7)}$$
$$+ A_t \left[\mu K_t^\rho + (1-\mu) K_{g,t}^\rho\right]^{\alpha/\rho} = 0$$

The first order conditions for new cars investment, miles driven and vehicle stock are given by:

$$\frac{\partial \mathcal{L}}{\partial X_t} : \quad -\vartheta_{1,t} p_t^X + \vartheta_{3,t} = 0, \quad \text{(B.8)}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} : \varphi_s \phi M_t^{\phi(1-\gamma)-1} Q_t^{1-\gamma} - \varphi_h \left(H_t + D_t\right)^{1/\upsilon} \eta_0 \eta_1 \frac{Q_t}{K_{g,t}} \left(\frac{M_t Q_t}{K_{g,t}}\right)^{\eta_1 - 1} \quad \text{(B.9)}$$
$$-\vartheta_{1,t} \left[\left(p_t^F \omega_F + p_t^{MR} \omega_{MR}\right) Q_t - \phi(1-\alpha)(1-\theta) \frac{Y_t}{M_t}\right] = 0,$$

$$\frac{\partial \mathcal{L}}{\partial Q_{t+1}} : \quad -\vartheta_{3,t} + \beta \mathbb{E}_t \Big[\vartheta_{3,t+1}\left(1-\delta_q\right) + \varphi_s M_{t+1}^{\phi(1-\gamma)} Q_{t+1}^{-\gamma} \quad \text{(B.10)}$$
$$-\varphi_h \left(H_{t+1} + D_{t+1}\right)^{1/\upsilon} \eta_1 \frac{D_{t+1}}{Q_{t+1}}$$
$$-\vartheta_{1,t+1} \left[\left[\left(p_{t+1}^F \omega_F + p_{t+1}^{MR} \omega_{MR}\right) M_{t+1} + p_{t+1}^{TI}\right] - (1-\theta)(1-\alpha) \frac{Y_{t+1}}{Q_{t+1}}\right]\Big] = 0.$$

The first order conditions for investment and capital stock are given by:

$$\frac{\partial \mathcal{L}}{\partial I_t} : -\vartheta_{1,t} + \vartheta_{2,t} = 0, \quad \text{(B.11)}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} : -\vartheta_{2,t} + \beta \mathbb{E}_t \left[\vartheta_{1,t+1} \frac{\alpha \mu K_{t+1}^\rho}{\mu K_{t+1}^\rho + (1-\mu) K_{g,t+1}^\rho} \frac{Y_{t+1}}{K_{t+1}} + \vartheta_{2,t+1}\left(1-\delta_k\right)\right] = 0. \quad \text{(B.12)}$$

The first order conditions for highway investment and road stock are given by:

$$\frac{\partial \mathcal{L}}{\partial I_{g,t}}: \quad -\vartheta_{1,t} + \vartheta_{4,t} = 0, \tag{B.13}$$

$$\frac{\partial \mathcal{L}}{\partial K_{g,t+1}}: \vartheta_{4,t} - \beta \mathbb{E}_t \Bigg[\varphi_h \left(H_{t+1} + D_{t+1}\right)^{1/v} \eta_1 \frac{D_{t+1}}{K_{g,t+1}} \tag{B.14}$$

$$+\vartheta_{1,t+1} \frac{\alpha \left(1-\mu\right) K_{g,t+1}^{\rho}}{\mu K_{t+1}^{\rho} + \left(1-\mu\right) K_{g,t+1}^{\rho}} \frac{Y_{t+1}}{K_{g,t+1}} + \vartheta_{4,t+1} \left(1-\delta_g\right) \Bigg] = 0.$$

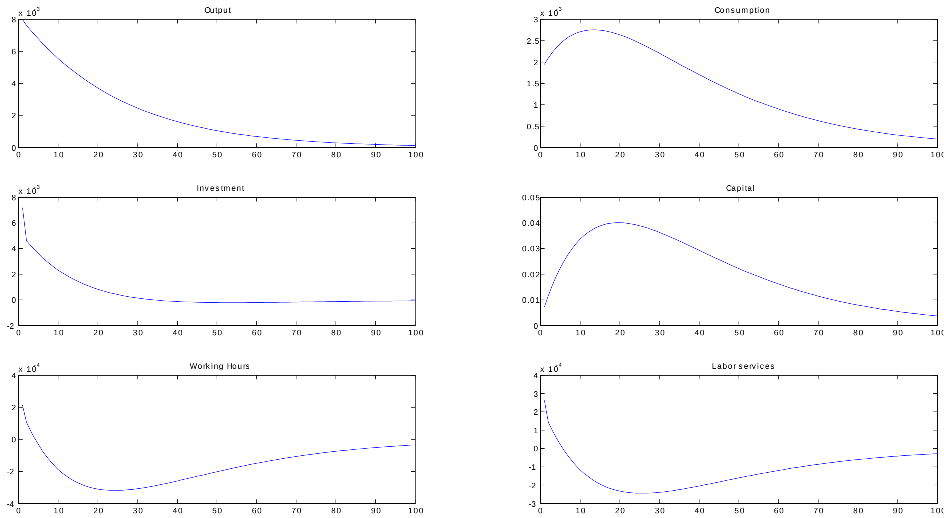

Figure 1: Total Factor Productivity shock (I)

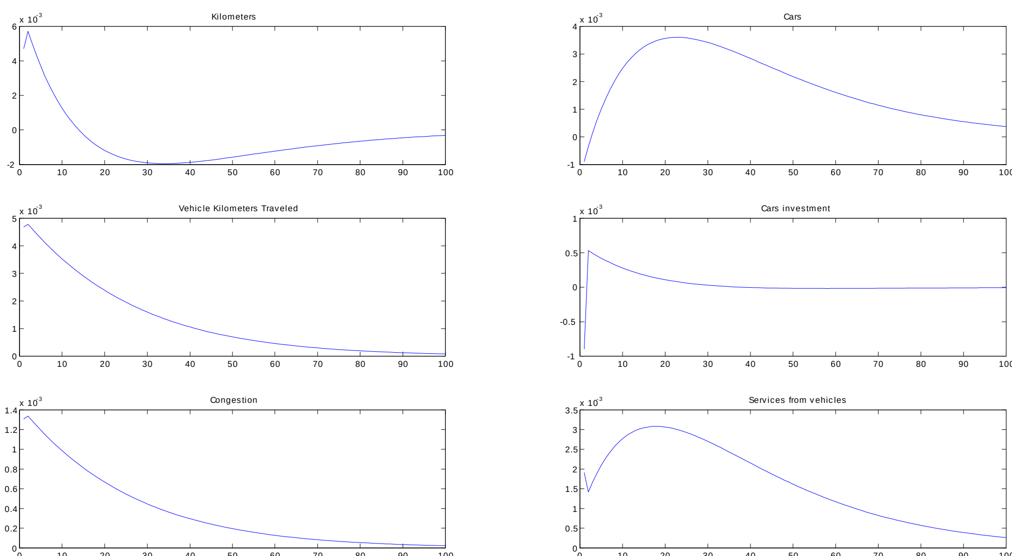

Figure 2: Total Factor Productivity shock (II)

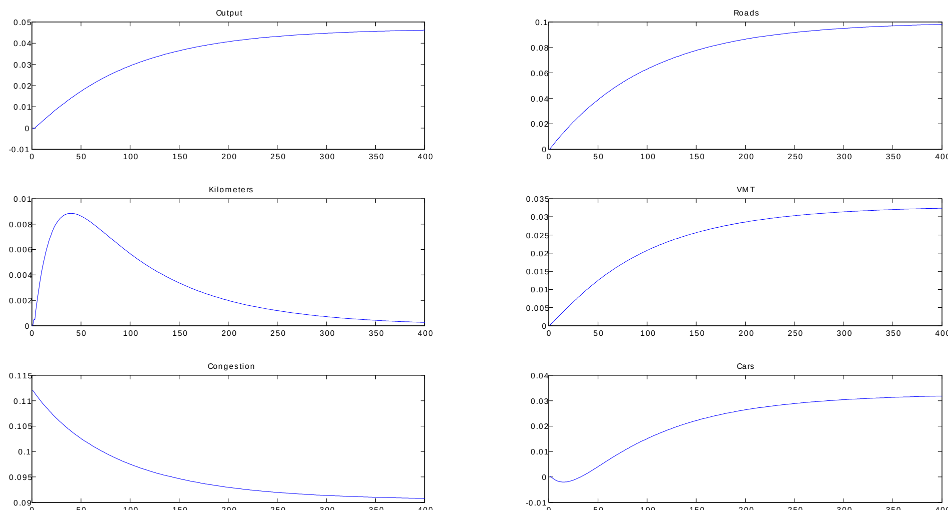

Figure 3: A rise in the stock of roads