ABSTRACTS OF THE TALKS

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CARLESON MEASURES FOR THE DRURY-ARVESON HARDY SPACE ON THE COMPLEX BALL

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Abstract

In this work in collaboration with R. Rochberg and E. Sawyer, we characterize the Carleson measures for the Drury-Arveson Hardy space, the space of the functions f, holomorphic on the unit ball $\mathbb{B}_n$ of $\mathbb{C}^n$, for which the Drury-ARveson norm $\|f\|_{DA}$ is finite,

$$\|f\|_{DA}^2 = \sum_{k=0}^{m-1} \left|f^{(k)}(0)\right|^2 + \int_{\mathbb{B}_n} \left|(1-|z|^2)^{m+1/2} f^m(z)\right|^2 \frac{dz}{(1-|z|^2)^{n+1}}.$$

Here, m is an integer such that $2(m+1/2) > n$ and dz is Lebesgue measure.

SPACEABILITY OF THE NON-EXTENDIBILITY

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Abstract

As introduced by Gurariy and Quarta (Bayart, resp.) at the beginning of the current millennium, a subset A of a topological vector space X is said to be spaceable (algebraically generic, resp.) whenever $A \cup \{0\}$ contains an infinite-dimensional closed linear manifold (a dense linear manifold, resp.) in X. These concepts become more interesting if A does not possess, a priori, a linear structure. With this aim, we consider the Fréchet space $H(G)$ of holomorphic functions in a domain G of the complex plane, endowed with the compact-open topology, and the subclass of $H_e(G)$ consisting of all functions having G as its domain of holomorphy. Aron, García and Maestre proved recently that $H_e(G)$ is both spaceable and algebraically generic. We show that this result can be strengthened by imposing growth conditions near the boundary. Moreover, under adequate conditions on G, the above assertion also holds for a number of function spaces of holomorphic functions

on G, different from $H(G)$. The results given here come partially from a joint work with M.C. Calderón and W. Luh.

NORM OF OPERATORS FROM HARDY INTO SEQUENCE SPACES

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Abstract

The objective is to give some estimates for the norm of operators $T; H^p \to \ell^q$ for different values of p, q in terms of certain norm in vector-valued $\ell_s(\ell_r)$ of the corresponding matrix coefficient.

WEIGHTED L^∞-ESTIMATES FOR BERGMAN PROJECTIONS

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Abstract

We report on joint work with Miroslav Engliš and Jari Taskinen.

We consider Bergman projections and some generalizations of them on weighted $L^\infty(D)$–spaces. A reproducing formula is obtained. We show the boundedness of these projections for a large family of weights v which tend to 0 at the boundary with a polynomial speed. These weights may even be nonradial. For logarithmically decreasing weights bounded projections do not exist. In this case we instead consider the projective description problem for weighted inductive limits of spaces of holomorphic functions on the disc.

BAKER DOMAINS FOR NEWTON'S METHOD

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Abstract

The Newton method for finding zeros of a function f consists of iterating the function

$$N(z) := z - \frac{f(z)}{f'(z)};$$

if z_0 is a root of f, then there is an N-invariant neightborhood U of Z_0 on which the iterates of N converge to z_0.

We consider N-invariant domains U on which the iterates N^k tend to ∞. Examples such as $f(z) = P(z) \exp Q(z)$ (with P, Q polynomials) show that there are 'Leau petals' at ∞ such that iterates of N tend to ∞ and $f \to 0$. Thus, the principle of Newton's rule still persists.

We present an example that there are entire functions for which N has an invariant domain on which $N^k \to \infty$, and yet 0 is not an asymptotic value. They may have any order in $(1/2, 1)$, and depend on the behavior of entire functions who grow very regularly, but whose zeros are distributed on a spiraling curve.

The talk is based on join work with W. Bergweiler and J. Langley.

SELF-IMPROVING BEHAVIOUR OF INNER FUNCTIONS AS MULTIPLIERS

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Abstract

For an inner function I, it is often true that multiplication by I either destroys smoothness completely or does not affect it at all. We shall discuss this phenomenon in various concrete situations.

FINITE INTERPOLATION ON THE CIRCLE

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Abstract

We consider the Nevanlinna-Pick problem on the unit circle $\partial\mathbb{D}$. That is, given n distinct points $z_1, \ldots, z_n \in \partial\mathbb{D}$ and n points (not necessarily distinct) $w_1, \ldots, w_n \in \partial\mathbb{D}$, find a function f such that $f(z_i) = w_i$ for all $i = 1, \ldots, n$.

It is known that the problem can always be solved by a Blaschke product of degree at most $n-1$. We will present an elementary algorithm for constructing interpolating Blaschke products that solve the problem, and which gives certain optimal solutions in some special cases.

A STRONG KIND OF CYCLIC OPERATORS: WEAKLY SUPERCYCLIC OPERATORS

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Abstract

A bounded linear operator T acting on a Banach space $\mathcal{B}$ is called weakly supercyclic if there is $x \in \mathcal{B}$ such that the projective orbit $\{\lambda T^n x : \lambda \in \mathbb{C}, n = 0, 1, \ldots\}$ is weakly dense in $\mathcal{B}$. If scalar multiples are not needed, then the operator is said to be weakly supercyclic. The corresponding (norm) supercyclic and (norm) hypercyclic concepts are well established. Due to a theorem of Mazur, weakly supercyclic operators are cyclic operators.

In this talk we will show several results on weakly weighted bilateral shifts. While hypercyclic and supercyclic bilateral shifts were characterized by Héctor Salas, characterizing weakly supercyclic or weakly hypercyclic bilateral weighted shifts seems to be much more subtle. Chan and Sanders have provided sufficient condition for a bilateral shift to be weakly hypercyclic. In this tallk we will show that a weighted bilateral shift, for $1 \leq p \leq 2$, is weakly supercyclic if and only if it is supercyclic. Also, the unweighted bilateral shift acting on $\ell^p(\mathbb{Z})$, $1 \leq p < \infty$, is weakly supercyclic if and only if $2 < p < \infty$. From the latter result, one immediately derives a celebrated result of Nikolski that determines in which ℓ^p the powers of the bilateral weighted shift are cyclic again. Another result, for any $p > 2$ there is a weakly hypercyclic, non-supercyclic bilateral weighted shift on $\ell^p(Z)$. For each $p \geq 2$ there is a bilateral weighted shift which is weakly hypercyclic on $\ell^p(\mathbb{Z})$ and not weakly hypercyclic on $\ell^r(\mathbb{Z})$ for each $r < p$. Thus, the minimal p for which there certain bilateral weighted shift is weakly hypercyclic takes all values between 2 and ∞.

If weakly supercyclicity is replaced by weak sequential density, then T is said to be weakly sequentially supercyclic. We can also show that on Hilbert space there are weakly supercyclic operators which are not weakly sequentially supercyclic. This is achieved by constructing a Borel probability measure μ on the unit circle in such a way that its Fourier coefficients vanish at infinity and the multiplication operator $(M_z f)(z) = zf(z)$ acting on $L^2(\mu)$ is weakly supercyclic, but it is not weakly sequentially supercyclic, since the projective orbit under M_z of each element in $L^2(\mu)$ is weakly sequentially closed. This answers a question posed by Bayart and Matheron.

Joint work with Stanislav A. Shkarin.

IMPROVEMENT OF ANALYTIC PROPERTIES UNDER THE LIMIT q-BERNSTEIN OPERATOR

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Abstract

The *limit q-Bernstein operator* $B_{\infty,q}$ on $C[0,1]$ emerges naturally as a limit of a sequence of q-Bernstein polynomials, $0 < q < 1$. In distinction from the classical case $q = 1$, the sequence of q-Bernstein polynomials for $q \in (0,1)$ does not satisfy the conditions of Korovkin's Theorem, and the limit q-Bernstein operator is *not* the identity operator. Recently, H. Wang proved a general Korovkin-type theorem which is applicable to sequences of q-Bernstein polynomials.

In this talk, we discuss the connection between the smoothness of f and the analytic properties of its image $B_\infty(f, q; x)$ with $q \in (0,1)$ being fixed. Our study reveals the following phenomenon: the limit q-Bernstein operator, in general, improves the analytic properties of functions. The improvement occurs for functions that are neither "too good" (polynomials) nor "too bad" (without a certain regularity condition).

For any $f \in C[0,1]$, a function $B_{\infty,q}f$ is analytic in $\{z : |z| < 1\}$. We discuss the conditions for analytic continuation of $B_\infty(f, q; x)$ into a disc $\{z : |z| < R\}$, where $R > 1$. It is shown that the smoother f at 1 is, the greater R becomes; and if f is infinitely differentiable at 1, then $B_\infty(f, q; z)$ is entire. We give estimates of growth for this entire function via magnitudes of consecutive derivatives of f.

It is shown that the smoothness properties of f can be recovered from the possibility of an analytic continuation for $B_{\infty,q}f$.

The talk contains new results as well as those known previously.

SPACES OF ANALYTIC FUNCTIONS OF HARDY-BLOCH TYPE

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Abstract

For $0 < p \leq \infty$ and $0 < q \leq \infty$, the space of Hardy-Bloch type $\mathscr{B}(p,q)$ consists of those functions f which are analytic in the unit disk $\mathbb{D}$ such that $(1-r)M_p(r,f') \in L^q(dr/(1-r))$. We note that $\mathscr{B}(\infty,\infty)$ coincides with the Bloch space $\mathscr{B}$ and that $\mathscr{B} \subset \mathscr{B}(p,\infty)$, for all p. Also, the space $\mathscr{B}(p,p)$ is the Dirichlet space $\mathcal{D}^p_{p-1}$.

We prove a number of results on decomposition of spaces with logarithmic weights which allow us to obtain sharp results about the mean growth of the $\mathscr{B}(p,q)$-functions. In particular, we prove that if f is an analytic function in $\mathbb{D}$ and $2 \leq p < \infty$, then the condition $M_p(r,f') = \mathrm{O}\left((1-r)^{-1}\right)$, as $r \to 1$, implies that $M_p(r,f) = \mathrm{O}\left(\left(\log \frac{1}{1-r}\right)^{1/2}\right)$, as $r \to 1$. This result is an improvement of the well known estimate of Clunie and MacGregor and Makarov about the integral means of Bloch functions, and it also improves the main result in a recent paper by Girela and Peláez. We also consider the question of characterizing the univalent functions in the spaces $\mathscr{B}(p,2)$, $0 < p < \infty$, and in some other related spaces and give some applications of our estimates to study the Carleson measures for the spaces $\mathscr{B}(p,2)$ and $\mathcal{D}^p_{p-1}$. The talk is based on joint work with D. Girela and M. Pavlovic

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RECURRENCE IN HYPERCYCLICITY

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Abstract

We discuss different notions of recurrence concerning the dynamics of linear operators $T : X \to X$. These notions are related to hypercyclicity and they can be expressed in terms of the sets of integers $N(x,U) := \{n \in \mathbb{N} \,;\, T^n x \in U\}$ and $N(U,V) := \{n \in \mathbb{N} \,;\, T^n(U) \cap V \neq \emptyset\}$, where $x \in X$ and $U, V \subset X$ are non-empty open sets. Some examples and applications are provided for weighted shifts on sequence spaces, translations operators on Hilbert spaces of entire functions, and solution semigroups of certain linear partial differential equations.

EXTENSIONS AND APPLICATIONS OF THE BEURLING-MALLIAVIN THEORY

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Abstract

Problems from several classical areas of analysis, such as completeness problems for sequences of reproducing kernels in Hilbert spaces of analytic functions, completeness problems for collections of special functions (Bessel, Airy, ...) in L2, spectral problems for Schroedinger and Krein string operators, etc. , can be reduced to questions about triviality of kernels of Toeplitz operators. Adopting the "Toeplitz approach" allows one to put all these areas into one picture, which helps find the right questions and select the best tools. In my talk I will try to illustrate this approach by discussing new generalizations and applications of the so-called Beurling-Malliavin theory. The talk is based on joint work with N. Makarov.

ISOMETRIC MULTIPLICATION AND DIVISION

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Abstract

For many classical function spaces, their pointwise self-multipliers are known, while the coefficient self-multipliers can be fully described only in the Hilbert space case. In this joint work with P. L. Duren and M. J. Martín we show that the multipliers (of either type) which are isometric (*i.e.*, norm-preserving) in several standard spaces can only be the trivial ones.

The isometric zero-divisors of Hardy spaces (for a given zero set) are well-known, while in Bergman spaces there are none. Although intuitively known to the experts since 30 years ago, the latter fact has only been proved rigorously in a 1993 paper by Duren, Khavinson, Shapiro, and Sundberg and in a complicated way (as a by-product of the development of the contractive zero-divisors via the positivity of the biharmonic Green function). We give a very short proof of the non-existence of isometric divisors for a general class of weighted spaces that satisfy certain axioms (among them a substitute for Fatou's lemma), based only on the above simple results on isometric multipliers.