ABSTRACTS OF THE COURSES

Winter School in Complex Analysis and Operator Theory
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A CLASS OF INTEGRAL OPERATORS ON SPACES OF ANALYTIC FUNCTIONS

Alexandru Aleman, Alexandru.Aleman@math.lu.se

Lund University, Sweden

Abstract

The lectures will be concerned with integral operators of the form

$$T_g f(z) = \int_0^z f(t)g'(t)dt$$

where g is a fixed analytic function in the unit disc called symbol. These operators appear in a natural way in many problems in complex analysis and I plan to give a detailed account on some of these. We shall then discuss various basic properties of these operators acting between some common spaces of analytic functions on the disc, more precisely between Hardy and weighted Bergman spaces. Most known results about these operators characterize the symbols g such that T_g acts as a bounded, compact or Schatten-class operator between given spaces as above and I will present some of these. Finally, we shall turn to more specific questions like the spectrum and invariant subspaces for such operators. These problems turn out to be quite difficult in the general case and for this reason we shall restrict our attention to certain special cases which are of interest in their own right.

DYNAMICS OF LINEAR OPERATORS

Karl Grosse-Erdmann, kg.grosse-erdmann@fernuni-hagen.de

University of Hagen, Germany

Abstract

In recent years a theory of linear dynamical systems has started taking shape. Building upon the simple notion of a hypercyclic operator, that is, an operator with a dense orbit, further concepts have been introduced and studied: chaotic operators, weakly mixing and mixing operators, and, most recently, frequently hypercyclic operators.

In the short course we shall first give an introduction to this emerging theory, stating its main definitions and presenting its main results. We shall, in particular, apply the theory to operators on spaces of analytic functions.

We shall then concentrate on recent investigations of F. Bayart and S. Grivaux who have applied methods from ergodic theory to the study of linear dynamical systems. The link between the two areas is provided by the notion of a Gaussian measure that is invariant for a given operator; properties of the invariant measure will then determine the dynamic behaviour of the operator.

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CLARK MEASURES, COMPOSITION OPERATORS, AND RELATED QUESTIONS

Eero Saksman, saksman@maths.jyu.fi

University of Jyväskylä, Finland

Abstract

Let ϕ be an analytic map mapping the complex unit disc $\mathbb{D}$ into itself. For each $\alpha \in \partial\mathbb{D}$ there corresponds a positive Borel measure τ_α on the boundary of the unit circle $\partial\mathbb{D}$ that is defined by means of the Poisson representation formula

$$\Re\frac{\alpha+\phi(z)}{\alpha-\phi(z)} = \int_{\partial\mathbb{D}} P(z,\zeta)\, d\tau_\alpha(\zeta).$$

Here P is the Poisson kernel for $\mathbb{D}$. The elements of the family of measures $\{\tau_\alpha\}_{\alpha\in\partial\mathbb{D}}$ are called spectral measures, or Clark measures (or Alexandrov-Clark measures), associated to the function ϕ. These measures have proven extremely useful in many questions of complex analysis or operator theory. Our purpose in these lectures is to investigate Clark measures and some of their applications in operator related function theory, starting from basic notions. Especially, we explore their central role in the theory of composition operators. We also investigate in more detail the connection between the Nevanlinna counting function of ϕ and the Clark measures, starting from Alexandrov's theory of values distribution of inner functions on the boundary ∂D. If time permits, we discuss some other applications, related to pseudocontinuation, generalized factorization, or perturbation theory.